第五屆培正數學邀請賽

5th Pui Ching Invitational Mathematics Competition

初賽（高中組）

Heat Event (Senior Secondary)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 設 $N=1001^4$。求 N 的數字之和。（3 分）

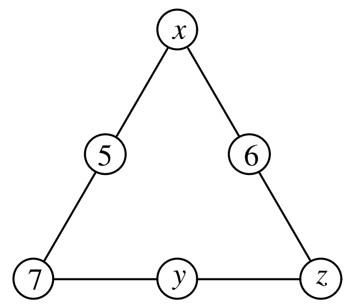
Let $N=1001^4$. Find the sum of the digits of N. (3 marks)

2. 小明把一些大於 2006 的質數乘起來，得到的答案爲 S。若 S 的個位數字是 T，問 T 有多少個不同的可能值？（3 分）

Mike multiplied together some prime numbers greater than 2006 and obtained the answer S. If the unit digit of S is T, how many different possible values of T are there? (3 marks)

3. 圖中，每個圓內都有一個數，使得三角形的每條邊上的三個圓內各數之和皆相同。求 z。（3 分）

In the figure, there is a number in each circle such that the sum of the numbers in the three circles on each side of the triangle is the same. Find z. (3 marks)



4. 方程 $x^2-2x-4=0$ 的兩個根是 $\tan m$ 和 $\tan n$，而方程 $kx^2+hx-1=0$ 的兩個根是 $\tan(90°-m)$ 和 $\tan(90°-n)$。求 k。（4 分）

The equation $x^2-2x-4=0$ has two roots $\tan m$ and $\tan n$ while the equation $kx^2+hx-1=0$ has two roots $\tan(90°-m)$ and $\tan(90°-n)$. Find k. (4 marks)

5. 若 x 是實數，且 $x^2-7x+10\le 0$，求 $x^2-6x+2006$ 的最大值。（4 分）

If x is a real number such that $x^2-7x+10\le 0$, find the maximum value of $x^2-6x+2006$. (4 marks)

6. 某人有超過一名兒子，他死後遺產按以下方法順序分配給他的兒子們。長子得到 100 元和餘下遺產的十分之一，次子得到 200 元和餘下遺產的十分之一，三子得到 300 元和餘下遺產的十分之一，如此類推。這樣，那人的遺產剛好由他的兒子們平分。該筆遺產的總數是多少元？（4 分）

A man has more than one son, and his legacy was distributed to his sons in the following order after he died. The eldest son got \$100 and one-tenth of the rest, the second son got \$200 and one-tenth of the rest, the third son got \$300 and one-tenth of the rest, and so on. In this way his legacy was equally divided among his sons. What is the amount of the legacy (in dollars)? (4 marks)

7. 設 n 爲大於 1 的整數。若 $n+\sqrt{n}+\sqrt[3]{n}$ 是整數，求 n 的最小可能值。（4 分）

Let n be an integer greater than 1. If $n+\sqrt{n}+\sqrt[3]{n}$ is an integer, find the smallest possible value of n. (4 marks)

8. 某圓形的方程是 $x^2+y^2+Ax+By+C=0$，其中 A、B、C 爲連續正整數，且 $A<B<C$。求 C 的最小可能值。（4 分）

The equation of a circle is $x^2+y^2+Ax+By+C=0$, where A, B, C are consecutive positive integers with $A<B<C$. Find the smallest possible value of C. (4 marks)

9. 已知對任意正整數 a，$\int_0^a kx^{2006}dx$ 皆是正整數。求 k 的最小可能值。（4 分）

It is known that for any positive integer a, $\int_0^a kx^{2006}dx$ is a positive integer. Find the smallest possible value of k. (4 marks)

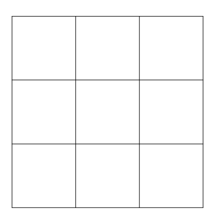
10. 如果某正整數 n 除以 23 時餘數比商大，則 n 稱爲「怪數」。那麼共有多少個「怪數」？（5 分）

A positive integer n is said to be ‘strange’ if, when n is divided by 23, the remainder is greater than the quotient. How many ‘strange’ integers are there? (5 marks)

11. 設 k 爲實數。若聯立方程 $\begin{cases} 4x-3y=3 \\ 6x+ky=4 \end{cases}$ 無解，求最接近 k^2 的整數。（5 分）

Let k be a real number. If the system of equations $\begin{cases} 4x-3y=3 \\ 6x+ky=4 \end{cases}$ has no solution, find the integer closest to k^2. (5 marks)

12. 圖中是一個 3×3 的方格表。現在我們要把每個方格都填上紅色、綠色或藍色，令每直行和橫行的三個方格的顏色都不相同。我們共有多少種填色方法？（5 分）



The figure shows a 3×3 grid. We want to colour each cell in red, green or blue in a way such that the three cells in each row and in each column receive different colours. How many different ways of colouring are there? (5 marks)

13. 在平面上畫出 10 條直線和 10 個圓形，最多可構成多少個交點？ （6 分）

If we draw 10 straight lines and 10 circles on the plane, what is the largest number of intersection points that can be formed? (6 marks)

14. 對於正整數 n，若把「123」連寫 n 次所得的正整數不可被 27 整除，則 n 稱爲「好數」。例如：因爲 123123 不能被 27 整除，所以 2 是「好數」。在不超過 2006 又不是「好數」的整數中，最大的一個是甚麼？ （6 分）

A positive integer n is said to be 'good' if the positive integer formed by writing '123' n times is **not** divisible by 27. For instance, 2 is 'good' since 123123 is not divisible by 27. Among the integers not exceeding 2006 which are **not** 'good', which is the largest one? (6 marks)

15. 某城市的電話號碼全爲八位數字，而所有電話的鍵盤款式均如圖所示。該城市並規定所有電話號碼的相鄰數字，必須爲電話鍵盤內的相鄰數字（例如：85256321）。若某電話號碼有四個不同的數字，求該電話號碼八個數字之和的最大可能值。 （6 分）

7	8	9
4	5	6
1	2	3

In a city, all telephone numbers have 8 digits and all telephones have a keyboard in the form as shown. Furthermore, consecutive digits in a telephone number must be adjacent digits on the keyboard (e.g. 85256321). If a telephone number consists of four different digits, find the greatest possible value of the sum of the 8 digits of the telephone number. (6 marks)

16. 已知 B 是正整數。小華做二次方程的練習題時，發現 $x^2+2Bx=0$、$x^2+2Bx+1=0$、$x^2+2Bx+2=0$、$x^2+2Bx+3=0$、…、$x^2+2Bx+12=0$ 這 13 條方程都有實數解，而且剛好有 3 條的兩個解都是整數。求 B 的值。 （6 分）

Given that B is a positive integer. When working on exercises in quadratic equations, Roy found that all 13 equations $x^2+2Bx=0$, $x^2+2Bx+1=0$, $x^2+2Bx+2=0$, $x^2+2Bx+3=0$, …, $x^2+2Bx+12=0$ have real roots and exactly 3 of them have both roots being integers. Find the value of B. (6 marks)

17. 某個奇數 n 有 123 個正因數。問 $4n$ 有多少個正因數？ （6 分）

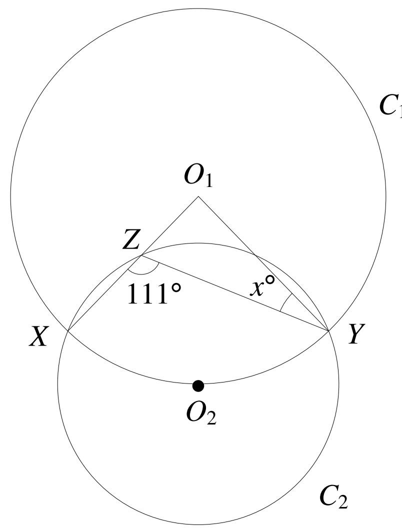
An odd number n has 123 positive factors. How many positive factors does $4n$ have? (6 marks)

18. 有兩個正整數，它們之和為 145。若把它們的最小公倍數除以它們的最大公因數，商是 168。求兩數之差。 （7 分）

Two positive integers have sum 145. When their L.C.M. is divided by their H.C.F., the quotient is 168. Find the difference between the two numbers. (7 marks)

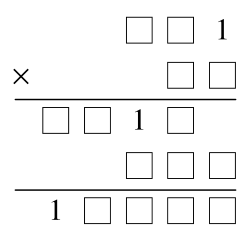
19. C_1 和 C_2 是兩個圓，圓心分別是 O_1 和 O_2，其中 O_2 位於 C_1 的圓周上。這兩個圓相交於 X 點及 Y 點。若 C_1 的半徑 O_1X 與 C_2 再相交於 Z，且 $\angle XZY = 111^\circ$、$\angle O_1YZ = x^\circ$，求 x。 （7 分）

C_1 and C_2 are two circles with centres O_1 and O_2 respectively, where O_2 lies on the circumference of C_1. These two circles intersect at the points X and Y. If the radius O_1X of C_1 intersects C_2 again at Z and $\angle XZY = 111^\circ$, $\angle O_1YZ = x^\circ$, find x. (7 marks)



20. 圖中顯示一條乘式，但當中有些數字留空了。若要使得乘積（即最底一行）最大，則乘積的最後四位數字應是甚麼？ （8 分）

The figure shows a multiplication, but some digits are left out. If the product (i.e. the last row) is to be largest, what should be the last four digits of the product? (8 marks)



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