

## 第五屆培正數學邀請賽

### 5th Pui Ching Invitational Mathematics Competition

#### 初賽（中三組）

#### Heat Event (Secondary 3)

**時限：1 小時 15 分**

**Time allowed: 1 hour 15 minutes**

**參賽者須知：**

**Instructions to Contestants:**

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 小明和小芬玩遊戲。小明持有四張分別寫上 1、4、5、8 的紙牌，小芬則持有四張分別寫上 2、3、6、7 的紙牌。兩人每次各出一張牌，數字較大者可得一分。每張牌只可出一次。小芬最多可得幾分？ (3 分)

Mike plays a game with Faye as follows. Mike has four cards with numbers 1, 4, 5, 8 respectively, while Faye has four cards with numbers 2, 3, 6, 7 respectively. Each time Mike and Faye each picks a card and the one whose number is larger gets 1 point. Each card can only be used once. What is the highest number of points that Faye may get? (3 marks)

2. 一個正方形的對角線的長度是 48。該正方形的面積是多少？ (3 分)

The diagonal of a square has length 48. What is the area of the square? (3 marks)

3. 若聯立方程  $\begin{cases} 3x + 2y = 10 \\ 2x + ky = 6 \end{cases}$  的解是  $x = 4$ 、 $y = -1$ ，求  $k$  的值。 (3 分)

If the system of equations  $\begin{cases} 3x + 2y = 10 \\ 2x + ky = 6 \end{cases}$  has solutions  $x = 4$  and  $y = -1$ , find the value of  $k$ . (3 marks)

4. 現有 16 盞燈，全都是關掉的。每盞燈有一個開關掣。如果按一次一盞關掉的燈上的開關掣，該盞燈會亮起；如果按一次一盞亮起的燈上的開關掣，該盞燈會關掉。若每天按 6 次開關掣，則最少要多少天才可使 16 盞燈全部亮起？ (3 分)

There are 16 lamps which are all turned off. There is a switch on each lamp. If a lamp is off and its switch is pressed once, the lamp will be turned on. If a lamp is on and its switch is pressed once, the lamp will be turned off. If we press switches 6 times a day, at least how many days are needed to turn on all the lamps? (3 marks)

5.  $A$  和  $B$  是平面上給定的兩點。若  $C$  是平面上滿足條件  $AB = AC$  和  $\angle BAC = 90^\circ$  的一點，那麼  $C$  的位置有多少個不同的可能性？ (3 分)

$A$  and  $B$  are two given points on the plane. If  $C$  is a point on the plane such that  $AB = AC$  and  $\angle BAC = 90^\circ$ , how many different possible positions for  $C$  are there? (3 marks)

6. 某人有超過一名兒子，他死後遺產按以下方法順序分配給他的兒子們。長子得到 100 元和餘下遺產的十分之一，次子得到 200 元和餘下遺產的十分之一，三子得到 300 元和餘下遺產的十分之一，如此類推。這樣，那人的遺產剛好由他的兒子們平分。該筆遺產的總數是多少元？ (4 分)

A man has more than one son, and his legacy was distributed to his sons in the following order after he died. The eldest son got \$100 and one-tenth of the rest, the second son got \$200 and one-tenth of the rest, the third son got \$300 and one-tenth of the rest, and so on. In this way his legacy was equally divided among his sons. What is the amount of the legacy (in dollars)? (4 marks)

7. 現有一個八重旋轉對稱圖形。若每次繞著旋轉對稱中心把圖形順時針旋轉  $12^\circ$ ，則最少要轉多少次才可使影像與原圖形重合？ (4 分)

There is a figure possessing rotational symmetry of order 8. If we rotate the figure clockwise by  $12^\circ$  through the centre of rotational symmetry each time, what is the minimum number of rotations needed so that the image will coincide with the original figure? (4 marks)

8. 若一件貨品以七折出售，售價會比原價低 420 元。若該貨品以六折出售，售價會比原價低多少元？ (4 分)

When an article is sold at a 30% discount, the selling price will be \$420 off the marked price. How many dollars will the selling price be off the marked price if the article is sold at a 40% discount? (4 marks)

9. 設  $x$ 、 $y$  為整數，且  $x^2 + y^2 = 2005$ 。求  $x$  所有可能值之和。 (5 分)

Let  $x$  and  $y$  be integers such that  $x^2 + y^2 = 2005$ . Find the sum of all possible values of  $x$ . (5 marks)

10. 某校中三級共有學生 220 名。在期中考試時，中文科合格的有 195 人、英文科合格的有 185 人、數學科合格的有 205 人，三科皆合格的有  $n$  人。求  $n$  的最大可能值與最小可能值之差。 (6 分)

In a certain school there are 220 students in Secondary 3. In the mid-year examination 195 students passed Chinese, 185 students passed English, 205 students passed Mathematics and  $n$  students passed all three subjects. Find the difference between the greatest and smallest possible values of  $n$ . (6 marks)

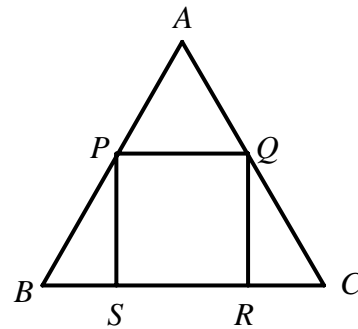
11. 對於正整數  $n$ ，若把「123」連寫  $n$  次所得的正整數不可被 27 整除，則  $n$  稱為「好數」。例如：因為 123123 不能被 27 整除，所以 2 是「好數」。在不超過 2006 又不是「好數」的整數中，最大的一個是甚麼？ (6分)

A positive integer  $n$  is said to be 'good' if the positive integer formed by writing '123'  $n$  times is not divisible by 27. For instance, 2 is 'good' since 123123 is not divisible by 27. Among the integers not exceeding 2006 which are not 'good', which is the largest one? (6 marks)

12. 在平面上畫出 10 條直線和 10 個圓形，最多可構成多少個交點？ (6分)

If we draw 10 straight lines and 10 circles on the plane, what is the largest number of intersection points that can be formed? (6 marks)

13. 圖中， $ABC$  是等邊三角形，邊長為 6。  $P$ 、 $Q$  分別是  $AB$ 、 $AC$  上的點， $R$ 、 $S$  是  $BC$  上的點，使得  $PQRS$  為正方形。求  $PQRS$  的面積，答案準確至最接近整數。



In the figure,  $ABC$  is an equilateral triangle of side length 6.  $P$ ,  $Q$  are points on  $AB$ ,  $AC$  respectively and  $R$ ,  $S$  are points on  $BC$  such that  $PQRS$  is a square. Find the area of  $PQRS$  correct to the nearest integer.

(6分)

(6 marks)

14. 某城市的電話號碼全為八位數字，而所有電話的鍵盤款式均如圖所示。該城市並規定所有電話號碼的相鄰數字，必須為電話鍵盤內的相鄰數字（例如：85256321）。若某電話號碼有四個不同的數字，求該電話號碼八個數字之和的最大可能值。

7	8	9
4	5	6
1	2	3

(6分)

In a city, all telephone numbers have 8 digits and all telephones have a keyboard in the form as shown. Furthermore, consecutive digits in a telephone number must be adjacent digits on the keyboard (e.g. 85256321). If a telephone number consists of four different digits, find the greatest possible value of the sum of the 8 digits of the telephone number.

(6 marks)

15. 在一次數學測驗中，每位學生的得分都是 0 至 100 的整數（包括 0 和 100），合格分數則為 40。某班的平均得分是 53。後來，一名得到 97 分的學生退學，結果餘下學生的平均得分變成 51。那麼，班中最多有幾人不合格？ (6分)

In a mathematics test, the score of each student is an integer between 0 and 100 (inclusive), and the passing score is 40. In a certain class, the average score of the students is 53. Later, a student who got 97 marks in the test withdrew from school and the average score of the remaining students of the class became 51. At most how many students in that class have failed the test? (6 marks)

16. 小華有 3000 個直徑為 1 cm 的圓形硬幣。他想用一些 1 cm 長的木棒圍成 3000 個邊長為 1 cm 的小正方形將這些硬幣一個一個地包圍著（這些小正方形可以有公共邊）。那麼，他最少要用多少枝木棒？ (6分)

Roy has 3000 circular coins of diameter 1 cm. He wants to use some wooden sticks which are 1 cm long to form 3000 squares with side length 1 cm in order to wrap each of the coins. (The squares can have common sides.) What is the least number of wooden sticks that he needs? (6 marks)

17. 圖中顯示一條乘式，但當中有些數字留空了。求乘積（即最底一行）的最後四位數字。 (6分)

The figure shows a multiplication, but some digits are left out. Find the last four digits of the product (i.e. the last row).

$$\begin{array}{r} \phantom{0000}9 \\ \times \phantom{0000}9 \\ \hline \phantom{0000}9 \\ \hline \phantom{0000}9 \end{array}$$

(6 marks)

18. 圖中所示為一個  $9 \times 3$  的方格陣，它被分成三個  $3 \times 3$  的方格陣。現要在每格填上 1 至 9 的其中一個數字，使得每個數字在各直行皆出現一次，而且每個數字在三個  $3 \times 3$  的方格陣中皆出現一次。那麼，有多少種方法填滿留空了的方格？ (6分)

The figure shows a  $9 \times 3$  grid, divided into three  $3 \times 3$  grids. Now we want to fill each cell with one of the numbers from 1 to 9. Each number must appear once in each column and each number must appear once in each of the three  $3 \times 3$  grids. How many different ways are there to fill the remaining squares? (6 marks)

1	4	7
2	5	8
3	6	9
4	7	1
5	8	2

(6分)

(6 marks)

19. 在一個神奇的罐子裏有 113 個紅球和 72 個綠球。罐子上有兩個按鈕甲和乙，只要按下它們便可以改變罐子裏球的數量。它們的作用如下：

按鈕甲：增加 4 個紅球和除去 5 個綠球；

按鈕乙：除去 6 個紅球和增加 5 個綠球。

如果按下某個按鈕時，某種顏色的球少於它可除去的球，則那種顏色的所有球會全被除去。假如我們希望罐子裏剛好有 60 個紅球和 60 個綠球，我們最少需要按下按鈕多少次？

(7 分)

In a magic can there are 113 red balls and 72 green balls. On the can there are two buttons, A and B. By pressing the buttons one can alter the number of balls in the can. The buttons work as follows:

Button A: Produce 4 red balls and remove 5 green balls

Button B: Remove 6 red balls and produce 5 green balls

If there are not enough balls of a certain colour to remove when a button is pressed, then the can will simply remove all balls of that colour. In order to get exactly 60 red balls and 60 green balls in the can, what is the minimum total number of times that the two buttons should be pressed?

(7 marks)

20. 設  $[x]$  代表不超過  $x$  的最大整數，例如  $[1.1] = 1$ 、 $[6.9] = 6$  和  $[5] = 5$ 。若  $[y] = 3$ 、 $[y^2] = 11$ 、 $[y^3] = 41$ ，問  $[y^6]$  有多少個不同的可能值？

(7 分)

Let  $[x]$  denote the greatest integer not exceeding  $x$ . For example,  $[1.1] = 1$ ,  $[6.9] = 6$  and  $[5] = 5$ . If  $[y] = 3$ ,  $[y^2] = 11$  and  $[y^3] = 41$ , how many different possible values of  $[y^6]$  are there?

(7 marks)

**全卷完**

**END OF PAPER**