

第五屆培正數學邀請賽

5th Pui Ching Invitational Mathematics Competition

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

The use of calculators is not allowed.

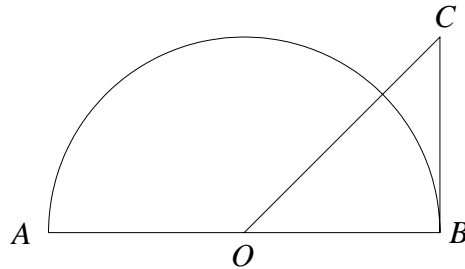
5. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 數列 1、2、4、8、16、... 除首項外，每項皆是之前一項的兩倍。若數列的第 2006 項是 2^k ，求 k 。 (3 分)

Except for the first term, each term of the sequence 1, 2, 4, 8, 16, ... is twice the previous term. If the 2006th term of the sequence is 2^k , find k . (3 marks)

2. 圖中， $\triangle OBC$ 是等腰直角三角形，其中 $BO = BC$ 。 AB 是以 O 為圓心的半圓的直徑。若 $AB = 24$ ，而全圖覆蓋範圍的面積是 $a + b\pi$ ，其中 a 和 b 是整數，求 $a + b$ 的值。 (3 分)



In the figure, $\triangle OBC$ is a right-angled isosceles triangle with $BO = BC$. AB is a diameter of the semi-circle with centre O . If $AB = 24$ and the figure covers an area of $a + b\pi$ where a and b are integers, find the value of $a + b$. (3 marks)

3. 小詩到時裝店買了一件襯衣和一條裙子。她有一張優惠券，憑券可以八折購買一件貨品。如果她在買襯衣時使用優惠券，她便需要付 230 元。如果她在買裙子時使用優惠券，她則只需付 220 元。襯衣的原價是多少元？ (4 分)

Tammy bought a blouse and a dress in a boutique. She had a coupon which entitled her to buy a piece of good with 20% discount. If she used the coupon when she bought the blouse, she had to pay 230 dollars. If she used the coupon when she bought the dress, she would only have to pay 220 dollars. What is the original price of the blouse (in dollars)? (4 marks)

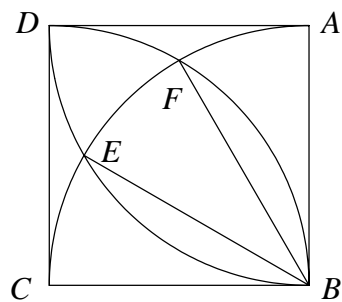
4. 小雄發現 3^{20} 的值是十位數 $x48678440y$ ，可是他忘記了這個數最左和最右的數字。求 $x + y$ 的值。 (4 分)

Peter found that the value of 3^{20} is a ten-digit number $x48678440y$. However, he forgot the leftmost and rightmost digits of the number. Find the value of $x + y$. (4 marks)

5. 小穎想了一個正整數 n ，並寫下了它所有小於 100 的正倍數。她發現在這些數當中，「8」這個數字出現了剛好四次。求 n 的值。 (4 分)

Diana thought of a positive integer n and wrote down all its positive multiples less than 100. She found that the digit '8' occurs exactly four times. Find the value of n . (4 marks)

6. 如圖所示， $ABCD$ 是正方形，而三段圓弧的圓心分別是 A 、 B 和 C ，半徑都與正方形的邊長相同。以 B 為圓心的圓弧分別交以 A 、 C 為圓心的圓弧於 E 和 F 。若 $\angle EBF = x^\circ$ ，求 x 。



(4 分)

As shown in the figure, $ABCD$ is a square. The centres of the three arcs are A , B and C while their radii are the same as the side length of the square. The arc centred at B meets the arcs centred at A and C at E and F respectively. If $\angle EBF = x^\circ$, find x .

(4 marks)

7. 在一個電視遊戲節目裏，100 名參賽者需於「紅」和「黃」之中二擇其一。他們作出選擇後仍有一次改變主意的機會。下表顯示了遊戲的統計數據，可是部分數字卻遺失了，只能以「？」顯示。根據表裏的資料，有多少名參賽者開始時選了「黃」，並且沒有改變主意？

(4 分)

In a TV game show, 100 contestants were asked to choose either 'red' or 'yellow'. After making the choice, they had a chance to change their options. The table below shows the statistics of the game. However, some data are missing, so they are only displayed with '?'. From the information in the table, how many contestants chose 'yellow' at first and did not change their options?

(4 marks)

第一次選擇 First option	紅 Red	?
	黃 Yellow	?
改變主意後 After the change of options	紅 Red	56
	黃 Yellow	?
改變主意的人數 Number of contestants changing their options		12
由紅轉黃的人數 Number of contestants changing from red to yellow		?
由黃轉紅的人數 Number of contestants changing from yellow to red		7

8. 小芳用了 7 塊長為 4、闊為 3 的長方形紙片拼成了一個大長方形。已知那些紙片沒有重疊，求大長方形的周界的可能值之和。

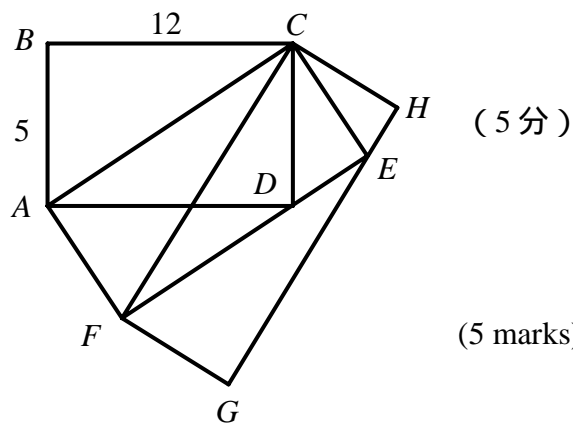
(5 分)

Amy used 7 pieces of rectangular cardboard with length 4 and width 3 to form a big rectangle. Given that there is no overlapping of cardboard, find the sum of all possible values of the perimeter of the big rectangle.

(5 marks)

9. 圖中 $ABCD$ 、 $ACEF$ 和 $CFGH$ 都是長方形，而 FDE 和 GEH 是直線。若 $AB=5$ 、 $BC=12$ ，求 $CFGH$ 的面積。

In the figure, $ABCD$, $ACEF$ and $CFGH$ are rectangles while FDE and GEH are straight lines. If $AB=5$ and $BC=12$, find the area of $CFGH$.



10. 平面上畫了五個圓和一些點。已知任何三個圓都沒有公共區域，而每個圓裏都有三點，那麼平面上最少畫了多少點？

Five circles and some points are drawn on the plane. Given that any three circles have no common area while every circle contains 3 points in its interior, at least how many points are drawn on the plane?

(5 分)

(5 marks)

11. 數列 1、4、9、16、... 把所有正平方數由小至大排列。如果將數列內所有個位數字為 4 的數刪掉後，第 2006 項便是 k^2 ，其中 k 是正整數。求 k 的值。

The sequence 1, 4, 9, 16, ... lists all positive square numbers in ascending order. If all terms with unit digit 4 are removed, the 2006th term is k^2 where k is a positive integer. Find the value of k .

(5 分)

(5 marks)

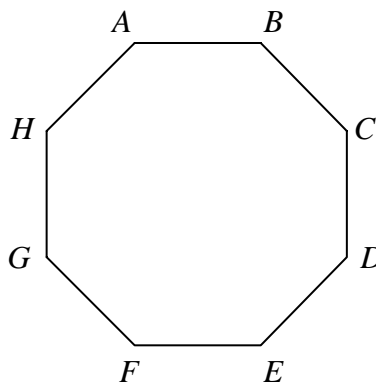
12. 小彤寫了一個共 2006 個正整數的數列。她發現由第 4 項開始，每一項都是它之前各項的平均值，而該數列的第 2006 項是 11。設數列首 3 項分別是 a 、 b 和 c ，求 $a+b+c$ 的值。

Eunice wrote a sequence with 2006 terms of positive integers. She discovered that from the 4th term onwards, every term is the average of its preceding terms while the 2006th term of the sequence is 11. Suppose that the first 3 terms of the sequence are a , b and c . Find the value of $a+b+c$.

(5 分)

(5 marks)

13. 小德和小培在圖中的正八邊形的跑道上跑步。他們同時在 A 點出發，並各自以均速沿順時針方向跑。他們在起跑後 1 小時於 D 點首次相遇。設於起跑後 k 小時他們首次在 A 點相遇，求 k 的值。



(5 分)

Sam and Tom ran on a track in the shape of a regular octagon. They started running from A and each ran with constant speeds along the clockwise direction. They first met at D one hour after they started running. Suppose they met at A for the first time k hours after they started running, find the value of k .

(5 marks)

14. 如果某正整數可以寫成兩個不同的合成數之和，我們便稱它為「好數」。例如，因為 $140 = 63 + 77$ ，而 63 和 77 是不同的合成數，所以 140 是「好數」。有多少個正整數不是「好數」？

(6 分)

A positive integer is said to be 'good' if it can be written as the sum of two distinct composite numbers. For instance, 140 is 'good' since $140 = 63 + 77$, where 63 and 77 are distinct composite numbers. How many positive integers are not 'good'?

(6 marks)

15. 在所示的算式中，每個字母代表一個 1 至 9 的不同數字。求 $A+B+C$ 的值。

In the calculation shown, each letter represents a different digit from 1 to 9. Find the value of $A+B+C$.

$$\begin{array}{r} A \ B \\ \times \quad A \\ \hline C \ B \ B \end{array}$$

(6 分)

(6 marks)

16. 一個問答比賽共有四隊參加。比賽共設 100 題，其中每題都只會由其中一隊答對。四隊的名次將按答對題數由多至少順序排列，假如答對題數相同，相關的隊伍便會抽籤決定名次。如果某隊要確保獲得第一名或第二名，他們最少要答對多少題？

(6 分)

Four teams compete in a quiz. There are 100 questions in the quiz while each question can be correctly answered by only one team. These four teams will be ranked in descending order of their numbers of correctly answered questions. In case of a tie, the ranks of the teams in question will be determined by drawing lots. If a team wants to guarantee to rank first or second, at least how many questions must they correctly answer?

(6 marks)

17. 在某保齡球三人賽裏，賽會規定了每隊的三名隊員的號碼之和必須是 10 的倍數。現有八名球員希望派出一隊參賽。如果他們的號碼分別是 15、22、33、43、55、65、72 和 82，他們的隊伍有多少個組合？ (6 分)

In a bowling trios, the organizing committee stated that the sum of the numbers of the three members of each team must be a multiple of 10. Eight players wanted to send a team to the trios. If their numbers were 15, 22, 33, 43, 55, 65, 72, 82, in how many ways could their team be formed? (6 marks)

18. 設 m 和 n 是兩個五位正整數，其中 $m > n$ 。已知 m 和 n 的 10 個數字都不相同。求 $m - n$ 的最小值。 (6 分)

Let m and n be two five-digit positive integers with $m > n$. It is known that the 10 digits of m and n are all different. Find the smallest possible value of $m - n$. (6 marks)

19. 某城市每 5 年的元旦便進行一次人口普查。王氏夫婦是該市的市民。他們二人在 1990 年結婚，婚後育有幾名孩子。已知在 1991 年、1996 年、2001 年和 2006 年的四次人口普查裏，王氏夫婦和他們的孩子的年齡總和皆是 8 的倍數。王氏夫婦二人最少有多少名孩子？（假設王家各人都活過 2006 年。） (7 分)

In a certain city, a census is conducted every 5 years on the New Year day. Wong's couple are citizens of that city. They got married in 1990 and gave birth to several children after marriage. It is known that in the four censuses in 1991, 1996, 2001 and 2006, the sums of the ages of the Wong's couple and their children are all multiples of 8. At least how many children does Wong's couple have? (Assume all members in the Wong's family are alive in 2006.) (7 marks)

20. 在圖中的 4×4 方格表裏，每個方格都填上了一個整數。包含了剛好一個平方數的長方形（包括正方形）有多少個？

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

In the figure, each square in the 4×4 square grid is labelled with an integer. How many rectangles (including squares) contain exactly one square number? (7 marks)

全卷完

END OF PAPER