

第四屆培正數學邀請賽
4th Pui Ching Invitational Mathematics Competition

初賽（中二組）
Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 某次數學比賽中，其中一題要求參賽者把某個不等於零的數除以 6。小芬誤把「除」字看作「乘」字。那麼，小芬的答案是正確答案的多少倍？

In a mathematical competition, a question asks contestants to divide a non-zero number by 6. Mary wrongly reads the word 'divide' as 'multiply'. How many times the correct answer is Mary's answer?

2. 現有八個方格，並要把每個塗上紅色或黃色。那麼至少有多少格的顏色相同？

There are 8 cells and each one is to be coloured red or yellow. What is the least number of cells which are of the same colour?



3. 求 1212、1818 和 2424 的最小公倍數。

Find the least common multiple of 1212, 1818 and 2424.

4. 2005 最大和最小的質因數相差多少？

What is the difference between the largest and the smallest prime factors of 2005?

第 5 至第 8 題，每題 4 分。

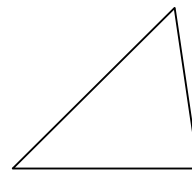
Questions 5 to 8 each carries 4 marks.

5. 在 2005 年中，有多少天的「月」和「日」均為質數？

In the year 2005, how many days are there such that the 'month' and the 'day' are both prime?

6. 現有一個三角形。至少要加上多少條直線，才能把三角形分成五個面積相等的部分？

A triangle is given. What is the minimum number of line segments that must be drawn in order to divide the triangle into 5 parts of equal area?



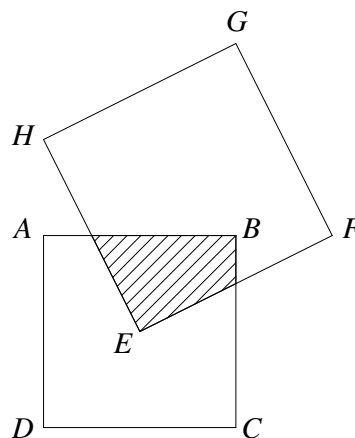
7. 小明在習作簿上寫上了一條正確的乘式，後來乘式中其中兩個個位數字卻被妹妹塗污了（見圖）。求兩個被塗污了的數字之積。

Alan put down a correct multiplication in his exercise book, and two unit digits in the multiplication were later crossed out by his sister, as in the figure. Find the product of the two digits which were crossed out.

$$\begin{array}{r} 15268 \blacksquare \\ \times \quad 9 \\ \hline 137418 \blacksquare \end{array}$$

8. 圖中， $ABCD$ 和 $EFGH$ 是正方形，邊長分別為 24 和 32。 E 是 $ABCD$ 的中心。求陰影部分的面積。

In the figure, $ABCD$ and $EFGH$ are squares with side lengths 24 and 32 respectively. E is the centre of $ABCD$. Find the area of the shaded region.



第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 某三角形三條邊的邊長分別是 2005、5002 和 n ，其中 n 是正整數。問 n 有多少個不同的可能值？

A triangle has side lengths 2005, 5002 and n , where n is a positive integer. How many different possible values of n are there?

10. 求最接近 $\sqrt{2+4+6+8+10+\cdots+2004}$ 的整數。

Find the integer closest to $\sqrt{2+4+6+8+10+\cdots+2004}$.

11. ABC 是等腰三角形， $\angle A = 68^\circ$ ， $\angle B = x^\circ$ 。求 x 所有可能值之和。

ABC is an isosceles triangle with $\angle A = 68^\circ$ and $\angle B = x^\circ$. Find the sum of all possible values of x .

12. 某班有 n 名學生。已知任何三名學生中，均最少有兩人的出生月份不同。求 n 的最大可能值。

There are n students in a class. It is known that among any three students, we can always find at least two who were born in different months of the year. Find the largest possible value of n .

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 對於正整數 n ，若 n 與 21 的最大公因數不是 1，那麼 n 便稱為「好數」。若把所有「好數」從小至大排列，第 2005 個是甚麼？

A positive integer n is said to be 'good' if the H.C.F. of n and 21 is NOT equal to 1. If all the 'good' numbers are arranged in ascending order, what is the 2005th term?

14. 設 $[x]$ 為不超過 x 的最大整數，例如 $[1.1] = 1$ 、 $[6.9] = 6$ 和 $[5] = 5$ 。若 n 是正整數， $\left[\frac{n}{2005}\right] \times \left[\frac{5002}{n}\right]$ 有多少個不同的可能值？

Let $[x]$ be the greatest integer not exceeding x . For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. If n is a positive integer, how many different possible values does $\left[\frac{n}{2005}\right] \times \left[\frac{5002}{n}\right]$ have?

15. 小強在一個凸 n 邊形內畫了 2005 條對角線，其中任何兩條在 n 邊形內部都不相交。求 n 的最小值。

Alan drew 2005 diagonals in a convex n -sided polygon, no two of which intersect in the interior of the n -sided polygon. Find the minimum value of n .

16. 我們有時會以「年 / 月 / 日」的形式表示日期。如果某天的「月」比「日」大，我們稱它為「美好的日子」。例如 2005/03/01 和 2005/04/03 都是「美好的日子」，2005/01/01 和 2005/03/12 則不是「美好的日子」。小芬只會在每個「美好的日子」看電視一次，其他日子都不會看。在 2005/02/01，她首次看電視。在 2025/11/09，她第 n 次看電視。求 n 。

We sometimes use the format 'Year/Month/Day' to represent dates. If the 'month' is greater than the 'day' in a day, we call it a 'nice day'. For example, 2005/03/01 and 2005/04/03 are 'nice days' while 2005/01/01 and 2005/03/12 are not. Sammy only watches television once during 'nice days' but not on other days. The first time she watched television was on 2005/02/01. On 2025/11/09, she will be watching television for the n -th time. Find n .

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 一個正方體的每個面上均寫有一個大於 1 的整數。若任何兩個相鄰的面上的兩數的最大公因數均為 1，求六個面上的六個數之和的最小可能值。

An integer greater than 1 is written on each face of a cube. Whenever two faces are adjacent, the H.C.F. of the two numbers on the faces is equal to 1. What is the minimum sum of the six numbers on the six faces?

18. 惠芬到偉強家中探望。偉強所居住的大廈的入口裝有密碼鎖，惠芬只知道密碼是一個四位正整數。她希望以最短的時間猜中密碼。她觀察到，密碼鎖的鍵盤上「3」、「6」、「9」這三個數字都褪了色，明顯是經常被按的，其他數字鍵則如新的一般，相信很少被按。她以此推斷，密碼由「3」、「6」、「9」這三個數字組成，而且每個數字都出現最少一次。假設惠芬的推斷正確，那麼在最壞情況下，她要試多少次才能開啓大門？

Betty visits Tommy at his home. The building in which Tommy lives has a password-operated lock at the main entrance. Betty only knows that the password is a four-digit positive integer. She hopes to guess the password in the shortest time. She observes that the numbers '3', '6' and '9' on the keypad appear old and seem to be frequently pressed, while the others appear new and seem to be seldom pressed. She therefore deduces that the password composes of the digits '3', '6' and '9' and each digit occurs at least once. Assuming that her deduction is correct, what is the number of trials she needs to make in the worst case before she can open the door?

19. 某多邊形的周界是 12，面積是 A 。該多邊形每邊的長度都是整數，而且每隻內角都是 90° 或 270° 。求 A 的所有可能值之和。

A polygon has perimeter 12 and area A . Each side of the polygon is an integer, and each of its interior angles is either 90° or 270° . Find the sum of all possible values of A .

20. 一隻鐳射影碟由三段片段組成，總長度為 47 分鐘。設該三段片段的片長分別為 a 分 b 秒、 c 分 d 秒和 e 分 f 秒，其中 a 、 b 、 c 、 d 、 e 、 f 都是小於 60 的整數，且 a 整除 b 、 c 整除 d 、 e 整除 f 。求 $\frac{b}{a} + \frac{d}{c} + \frac{f}{e}$ 的最大可能值。

A videodisc comprises 3 episodes with total length 47 minutes. If the lengths of the 3 episodes are a minute(s) b second(s), c minute(s) d second(s) and e minute(s) f second(s) respectively, where a, b, c, d, e, f are all integers less than 60 such that a divides b , c divides d and e divides f , find the maximum possible value of $\frac{b}{a} + \frac{d}{c} + \frac{f}{e}$.

全卷完

END OF PAPER