

**第四屆培正數學邀請賽**  
**4th Pui Ching Invitational Mathematics Competition**

**決賽（中四組）**  
**Final Event (Secondary 4)**

**時限：2 小時**

**Time allowed: 2 hours**

**參賽者須知：**

**Instructions to Contestants:**

1. 本卷共設甲、乙兩部分，總分爲 100 分。  
This paper is divided into Section A and Section B. The total score is 100.
2. 除特別指明外，本卷內的所有數均爲十進制。  
Unless otherwise stated, all numbers in this paper are in decimal system.
3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。  
Unless otherwise stated, all answers should be given in exact numerals in their simplest form.  
No approximation is accepted.
4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。  
Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.
5. 不得使用計算機。  
The use of calculators is not allowed.
6. 本卷的附圖不一定依比例繪成。  
The diagrams in this paper are not necessarily drawn to scale.

## 甲部 (60分)

### Section A (60 marks)

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

第 5 至第 8 題，每題 5 分。

Questions 5 to 8 each carries 5 marks.

第 9 至第 12 題，每題 7 分。

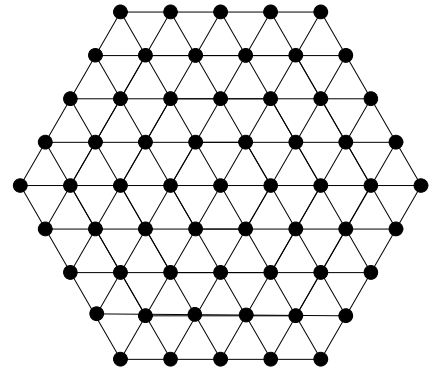
Questions 9 to 12 each carries 7 marks.

1. 一張大小為  $24 \times 32$  的長方形紙張可以剪成  $n$  小塊，使得每小塊皆是正方形。(每小塊的大小不一定相同。) 求  $n$  的最小可能值。

A rectangular piece of paper of size  $24 \times 32$  can be cut into  $n$  small pieces such that each small piece is square in shape. (The small pieces are not necessarily of the same size.) Find the smallest possible value of  $n$ .

2. 圖中的 61 點排列成 96 個面積為 1 的等邊三角形。若在這 61 點中選取 4 點，使其成爲一個四邊形的頂點，這個四邊形的面積最大是多少？

The 61 points in the figure make up 96 equilateral triangles each with area 1. If 4 points are to be chosen from these 61 points so that they form the vertices of a quadrilateral, what is the largest possible area of this quadrilateral?



3. 已知  $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$ 。把  $58 \times 72$  寫成兩個平方數之和。(答案以  $x^2 + y^2$  形式表示，其中  $x$ 、 $y$  爲正整數。)

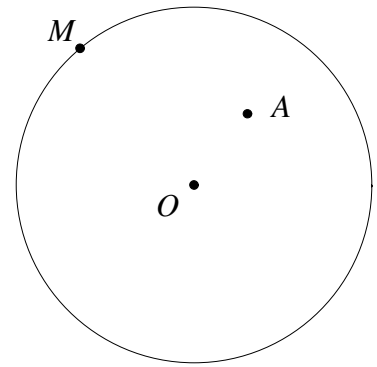
It is known that  $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$ . Express  $58 \times 72$  as the sum of two square numbers. (Express your answer in the form  $x^2 + y^2$ , where  $x$  and  $y$  are positive integers.)

4. 圖中所示的是一個幻方，它每個橫行、每個直行和每條對角線上三個數之和都是相同的。然而，幻方中有些方格留空了。左下角那一格的數應該是甚麼？

The figure shows a magic square in which the sum of the three numbers in each of its rows, columns and diagonals is the same. However, some entries are missing. What should be the number in the lower left hand corner?

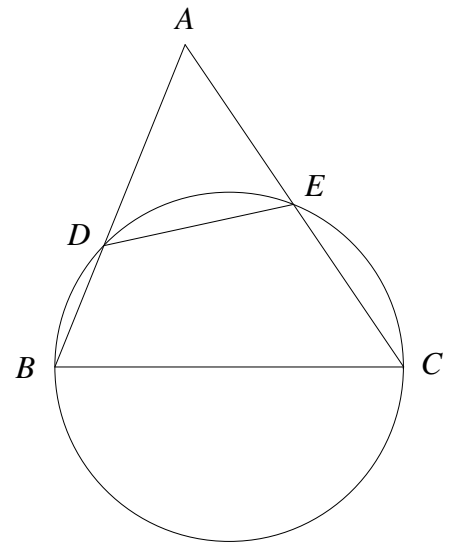
13	4	
		8
		9

5. 圖中， $O$  是圓的圓心，圓半徑為 3。 $A$  是圓內的一點，使得  $AO = 1$ 。若  $M$  是圓周上的一點，使得  $\angle AMO$  最大，求  $\sin \angle AMO$ 。



In the figure,  $O$  is the centre of the circle. The radius of the circle is 3.  $A$  is a point inside the circle with  $AO = 1$ .  $M$  is a point on the circumference such that  $\angle AMO$  is the greatest. Find  $\sin \angle AMO$ .

6. 圖中， $\angle BAC = 30^\circ$ ， $BC$  為圓的直徑， $AB$ 、 $AC$  分別與圓相交於  $D$ 、 $E$ 。若  $\triangle ABC$  的面積是 12，求四邊形  $BCED$  的面積。



In the figure,  $\angle BAC = 30^\circ$ ,  $BC$  is a diameter of the circle and  $AB$ ,  $AC$  at meet the circle at  $D$ ,  $E$  respectively. If the area of  $\triangle ABC$  is 12, find the area of the quadrilateral  $BCED$ .

7. 小明和小麗分別就讀於甲、乙兩所學校。某天，甲校其中 42% 的學生轉往乙校，結果乙校的學生人數比甲校多 56%。求乙校原有的學生人數的最小值。

Myron and Lily study in Schools A and B respectively. One day, 42% of the students of School A were transferred to School B. As a result, School B has 56% more students than School A. What is the smallest possible value of the original number of students in School B?

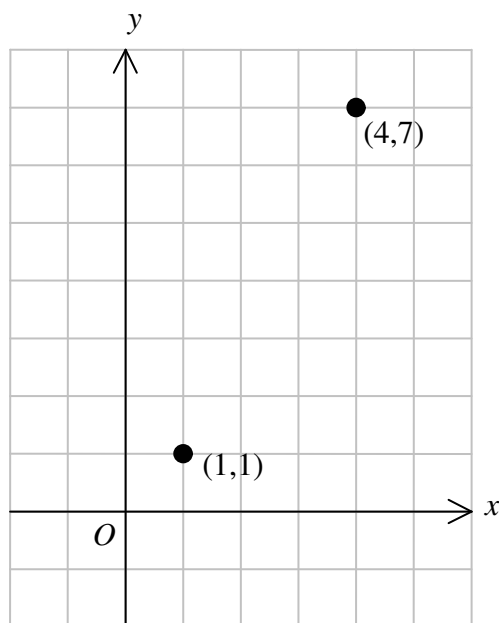
8. 在所示的算式中，不同的字母代表不同的數字。已知  $S = 5$ ，求四位數 MATH 的值。

$$\begin{array}{r}
 \text{M A T H} \\
 \text{A T} \\
 \text{P C M S} \\
 + \qquad \qquad \qquad 9 \\
 \hline
 \text{A P R I L}
 \end{array}$$

In the calculation shown, different letters represent different digits. Given that  $S = 5$ , find the value of the four-digit integer MATH.

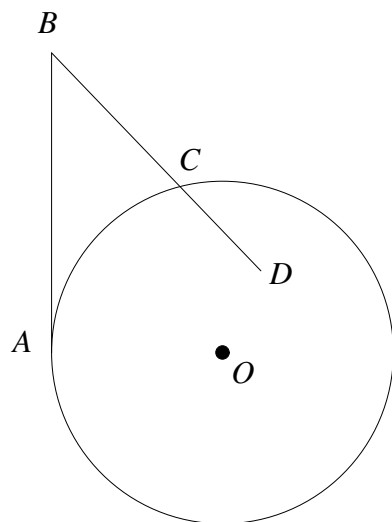
9. 在直角坐標平面上，一隻螞蟻從 (1,1) 出發，每步皆向上、下、左或右走 1 單位。走了 11 步後，螞蟻在 (4,7) 停了下來。螞蟻所走的路線有多少個不同的可能？

On the rectangular coordinate plane, an ant starts walking from the point (1,1). In each step it walks 1 units upwards, downwards, to the left or to the right. After 11 steps it stops at the point (4,7). How many different possibilities are there for the route taken by the ant?



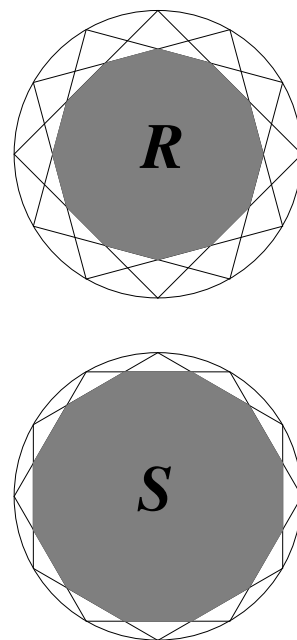
10. 如圖所示， $O$  是圓的圓心， $AB$  切圓於  $A$ ， $D$  是圓內的一點， $DB$  與圓相交於  $C$ 。若  $BC = CD = 3$ ， $OD = 2$ ， $AB = 6$ ，求圓的半徑。

In the figure,  $O$  is the centre of the circle,  $AB$  is tangent to the circle at  $A$ ,  $D$  is a point in the circle and  $DB$  intersects the circle at  $C$ . If  $BC = CD = 3$ ,  $OD = 2$  and  $AB = 6$ , find the radius of the circle.



11. 如圖所示，兩人各自在兩個全等的圓形的圓周上標了 12 點將圓周分成 12 等份，再分別用不同的方法連起這些點，最後再把圖案中央的正十二邊形填色。若正十二邊形  $R$  和  $S$  的邊長分別是  $r$  cm 和  $s$  cm，求  $\frac{r}{s}$ 。

As shown in the figure, two people each marks 12 points on the circumference of two identical circles to divide the circumference into 12 equal parts. After that, they connect the points in different ways and then colour the regular 12-sided polygon in the middle. If the side lengths of regular 12-sided polygons  $R$  and  $S$  are  $r$  cm and  $s$  cm respectively, find  $\frac{r}{s}$ .



12. 區先生和五位學生小陳、小李、小張、小王和小何玩遊戲。他準備了 7 張分別寫上了 1、2、3、4、5、6 和 7 的咭片，並在每位學生額前都放一張咭，以及藏起餘下的兩張咭。每人都只可看到別人額前的咭上的數字，但看不見自己額前的咭上的數字。以下是他們於較後時間的對話。

區先生問：「小陳，你知道自己額前的數字嗎？」

「我不知道，但我知道我的額前的數字大於小李和小張的數字。」小陳回答。

區先生問所有學生：「現在有人知道自己額上的數字嗎？」

小李、小張和小王一同答道：「我知道了。」

「可是我們還沒有知道呢。」小陳和小何投訴。

「讓我給你們一點提示吧。」小李說。「你們額前的數字之和是 4 的倍數。」

小陳和小何齊回答：「我們知道了。謝謝小李。」

假設學生都是聰明的（即當他們有足夠的資料便一定可以作出推論），求兩張藏起了的咭上的數字之積。

Mr Au played a game with five students, Ben, Carl, Don, Eric and Fred. He prepared 7 cards with the numbers 1, 2, 3, 4, 5, 6 and 7 written respectively. Mr Au put a card on the forehead of each student. The two remaining cards are hidden. Every student could see the numbers on the cards on others' forehead but not the one on his. Their subsequent conversations are as follows.

'Ben, do you know the number on your forehead?' Mr Au asked.

'No, but I know the number on my forehead is greater than Carl's and Don's,' said Ben.

'Does anyone know the number on your forehead now?' Mr Au asked all students.

'I know.' replied Carl, Don and Eric together.

'But we still do not know what they are.' Ben and Fred complained.

'Let me give you a hint.' said Carl, 'the sum of the numbers on your foreheads is a multiple of 4.'

'We know now. Thanks, Carl.' answered Ben and Fred.

Assume that all students are intelligent (i.e. they can make deductions whenever there is enough information). What is the product of the hidden numbers?

乙部 (40 分)

Section B (40 marks)

13. 一位外星人帶了一部特別的鋼琴來訪地球。這台鋼琴有 2005 個琴鍵，分別編號為 1 號、2 號、...、2005 號。連續的琴鍵之間的距離相同。

An alien visited the Earth with his special piano. This piano has 2005 keys, which are labelled as number 1, number 2, ..., number 2005. Consecutive keys are separated by the same distance.

1	2	3	...	...	...	...	2004	2005
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- (a) 外星人每次按下剛好  $k$  個琴鍵，按了 101 次後，就把每個琴鍵都按了奇數次（不同琴鍵被按的次數不一定相同）。求  $k$  最小的可能值。 (5 分)

The alien pressed exactly  $k$  keys each time for 101 times. As a result of this, each key has been pressed an odd number of times (different keys may be pressed by different numbers of times). What is the smallest possible value of  $k$ ? (5 marks)

- (b) 外星人有 101 隻手指。當他每次以他的 101 隻手指同時按下 101 個不同的琴鍵，且任何兩隻相鄰的手指的距離皆相同時，則我們稱這 101 個琴鍵時所發出的音為一個「和弦」。這特別的鋼琴可以奏出多少個不同的「和弦」？ (5 分)

The alien has 101 fingers. When it uses all its 101 fingers to press 101 different keys simultaneously in a way such that the distances between successive fingers are the same, then we call the sound obtained by pressing these 101 keys a 'chord'. How many 'chords' can this special piano produce? (5 marks)

- (c) 爲了表示友好，外星人按下了  $m$  號及  $n$  號琴鍵，其中  $(m^2, n^2) = 6(m, n) - 9$ ，這裡  $(x, y)$  代表  $x$  和  $y$  的最大公因數。求  $m+n$  的最大可能值。 (6分)

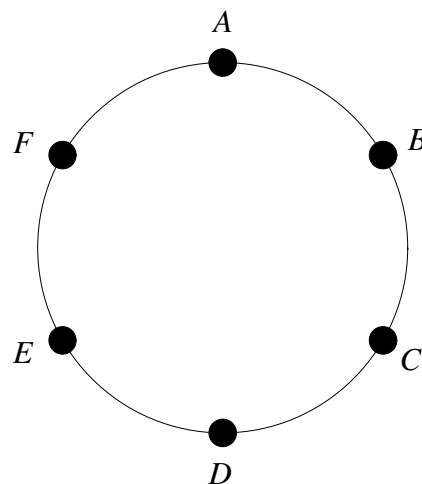
To say hello, the alien pressed two keys which are labelled as number  $m$  and number  $n$ , where  $m$  and  $n$  satisfy  $(m^2, n^2) = 6(m, n) - 9$ . Here  $(x, y)$  denotes the H.C.F. of  $x$  and  $y$ . Find the greatest possible value of  $m+n$ . (6 marks)

- (d) 外星人離開地球前，送了一塊寶石給地球人。這寶石的形狀是一個長方體，大小爲15厘米×30厘米×75厘米。地球人製作了多個寶石的複製本作展覽之用。每一個複製本均與寶石實物相似，且有一邊長10厘米。複製本的體積的所有可能值之和是多少立方厘米？ (4分)

The alien gave us a gemstone as a present when he left the Earth. The gemstone is in the shape of a cuboid and its dimensions are 15cm×30cm×75cm. A number of copies of the gemstone were made for display purposes. Each copy is similar to the original gemstone, and has a side of length 10 cm. Find, in  $\text{cm}^3$ , the sum of all possible volumes of the gemstone copies. (4 marks)

14. 如圖所示，六個人  $A$ 、 $B$ 、 $C$ 、 $D$ 、 $E$  和  $F$  依次序圍成一個半徑爲1的圓玩遊戲，任何相鄰的兩人之間的距離皆相同。裁判把六張分別寫上1至6的遊戲卡分派給六人。每個人只可以看到自己和他旁邊兩人的卡上的數字。每個人所得的分數就是他所看到的三張卡上的數字之積。

Six people,  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$  and  $F$ , sit around a circle of radius 1 as shown in the figure. Any two adjacent players are separated by the same distance. Six cards numbered 1 to 6 are distributed to the six people by a judge. Each person can only see the numbers on his own card and the cards of his two neighbours. The score of each person is the product of the three numbers he can see.



- (a)  $A$  的得分有多少個不同的可能值？ (4 分)  
How many different possible values are there for the score of  $A$ ? (4 marks)

- (b) 求六人得分總和的最大可能值。 (6 分)  
Find the greatest possible value of the total score of the six people. (6 marks)

後來裁判隨意把六人分成兩組，每組各三人。每組的三位成員都可連成一個三角形，而裁判必須坐在兩個三角形的重疊位置。若兩個三角形沒有重疊，則須重新分組。

The judge then randomly divides the six people into two groups of three. The three people in each group form a triangle, and the judge must sit in the area common to the two triangles. If the two triangles do not overlap, the judge will need to regroup the six people.

- (c) 求需要重新分組的概率。 (5 分)  
Find the probability that regrouping is necessary. (5 marks)

- (d) 結果， $A$ 、 $C$ 、 $D$  被編進一組， $B$ 、 $E$ 、 $F$  則被編進另一組。求裁判可坐的區域的面積。 (5 分)  
In the end,  $A$ ,  $C$ ,  $D$  form a group and  $B$ ,  $E$ ,  $F$  form another group. Find the area of the region in which the judge may sit. (5 marks)

**全卷完**

**END OF PAPER**