

**第四屆培正數學邀請賽**  
**4th Pui Ching Invitational Mathematics Competition**

**決賽（中二組）**  
**Final Event (Secondary 2)**

**時限：2 小時**

**Time allowed: 2 hours**

**參賽者須知：**

**Instructions to Contestants:**

1. 本卷共設甲、乙兩部分，總分爲 100 分。  
This paper is divided into Section A and Section B. The total score is 100.
2. 除特別指明外，本卷內的所有數均爲十進制。  
Unless otherwise stated, all numbers in this paper are in decimal system.
3. 除特別指明外，所有答案須以數字之真確值表達，並化至最簡。不接受近似值。  
Unless otherwise stated, all answers should be given in exact numerals in their simplest form.  
No approximation is accepted.
4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。  
Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.
5. 不得使用計算機。  
The use of calculators is not allowed.
6. 本卷的附圖不一定依比例繪成。  
The diagrams in this paper are not necessarily drawn to scale.

甲部 (60分)

Section A (60 marks)

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

第 5 至第 8 題，每題 5 分。

Questions 5 to 8 each carries 5 marks.

第 9 至第 12 題，每題 7 分。

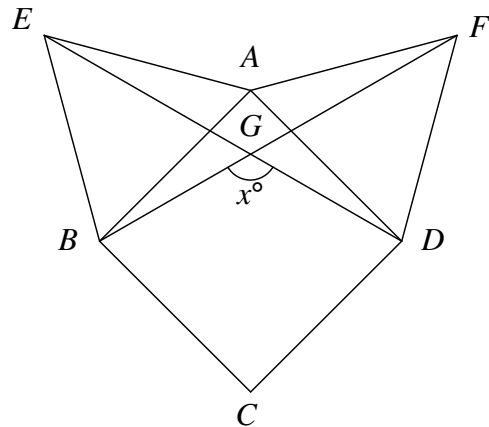
Questions 9 to 12 each carries 7 marks.

1. 一張大小為  $16 \times 24$  的長方形紙張可以剪成  $n$  小塊，使得每小塊皆是正方形。(每小塊的大小不一定相同。) 求  $n$  的最小可能值。

A rectangular piece of paper of size  $16 \times 24$  can be cut into  $n$  small pieces such that each small piece is square in shape. (The small pieces are not necessarily of the same size.) Find the smallest possible value of  $n$ .

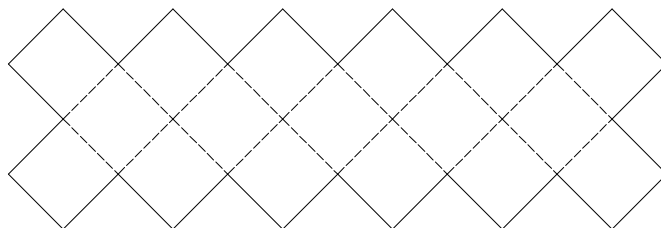
2. 圖中， $ABCD$  為正方形， $ABE$  和  $ADF$  為等邊三角形， $BF$  交  $DE$  於  $G$ 。若  $\angle BGD = x^\circ$ ，求  $x$ 。

In the figure,  $ABCD$  is a square while  $ABE$  and  $ADF$  are equilateral triangles.  $BF$  and  $DE$  meet at  $G$ . If  $\angle BGD = x^\circ$ , find  $x$ .



3. 下圖是一種特別的棋盤。該棋盤由 17 個面積為 1 的正方形組成。這種棋的棋子都是  $1 \times 2$  的長方形，而且必須放在棋盤上兩個相鄰的方格內。如果棋子不容許重疊，那麼棋盤上最多可放多少隻棋子？

The figure below shows a special type of chessboard. The chessboard is made up of 17 squares of area 1. The chess pieces are all  $1 \times 2$  rectangles, and must be placed on two consecutive squares of the chessboard. If overlapping is not allowed, what is the maximum number of pieces of chess that can be placed on the chessboard?



4. 以下是一段清朝的歷史：

「乾隆晚年，寵信和珅，和珅則借機貪污。這個和珅竟想打皇上的貢品主意！每有貢品，和珅都先扣起一部分，才將餘下的呈交皇上。而皇上又會將所呈交的貢品的四成賜予和珅。結果，和珅實際所得的貢品，竟比皇上實際所得的多一半！」

問和珅所扣起的「一部分」，佔原有貢品的幾分之幾？

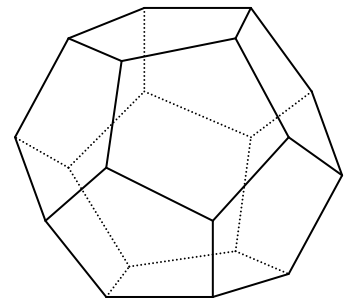
Here is a piece of history of the Qing dynasty:

*“Emperor Qian-long trusted his official He-shen. He-shen took advantage of this to corrupt. Whenever there were gifts to the emperor, he would keep a part of the gifts himself and pass the remaining gifts to the emperor. The emperor then gave 40% of the remaining gifts to He-shen. The gifts obtained by He-shen turned out to be half more than those actually obtained by the emperor!”*

What fraction of the original gifts was the ‘part of the gifts’ kept by He-shen before passing the rest to the emperor?

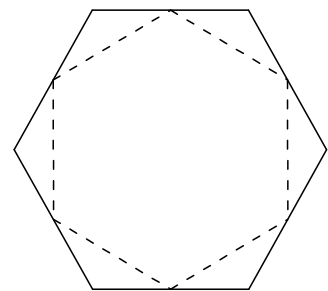
5. 一隻螞蟻在一個正十二面體的其中一個頂點出發，沿著它的邊走。如果該螞蟻不走重覆的路，且最後回到它的出發點，那麼該螞蟻最多可走過多少條邊？

On a regular dodecahedron, an ant starts at a vertex and walks along its edges. If the ant does not repeat its path and eventually returns to the starting point, what is the maximum number of edges that the ant can go through?



6. 現有一個正六邊形。若把六邊的中點如圖連起，可得一個較小的正六邊形。小六邊形的面積是大六邊形的面積的幾分之幾？

There is a regular hexagon. By joining the mid-points of the six sides as shown, we get a smaller regular hexagon. What fraction of the area of the original hexagon is the area of the small hexagon?



7. 在所示的算式中，不同的字母代表不同的數字。已知  $S = 2$ 、 $T = 4$ ，求四位數 MATH 的值。

In the calculation shown, different letters represent different digits. Given that  $S = 2$  and  $T = 4$ , find the value of the four-digit integer MATH.

$$\begin{array}{r}
 P \quad C \quad M \quad S \\
 M \quad A \quad T \quad H \\
 \phantom{M \quad A} \quad S \quad A \quad T \\
 + \phantom{M \quad A \quad S} \phantom{M \quad A} \phantom{M \quad A} \quad 9 \\
 \hline
 A \quad P \quad R \quad I \quad L
 \end{array}$$

8. 三所學校被邀請出席一個大型活動。主辦單位安排了旅遊車接載參加活動的師生，而每輛旅遊車的載客量都是相同的。甲校有 53 名師生出席，需要 4 輛旅遊車；乙校有 113 名師生出席，需要 7 輛旅遊車。若丙校有 2005 名師生出席，需要多少輛旅遊車？

Three schools are invited to attend a mass function. The organizers have arranged coaches to send students and teachers to the venue. All coaches carry the same number of passengers. There are 53 participants in School A and 4 coaches are needed. There are 113 participants in School B and 7 coaches are needed. If there are 2005 participants in School C, how many coaches are needed for School C?

9. 小華和小明玩遊戲：現有兩個袋子，它們分別有 1200 枚和 1800 枚硬幣。兩人按以下的規則輪流取去硬幣：

- 每人每次只可在其中一個袋子裡取硬幣，最少取走 1 枚硬幣，但不可把袋中所有硬幣取走。
- 每次取完硬幣後，必須有至少一個袋子中的硬幣數量是偶數。

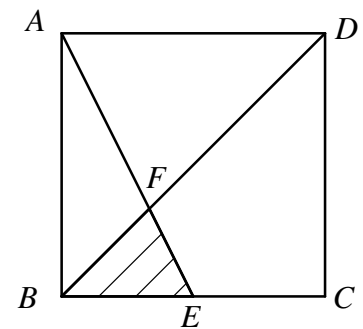
若某人不能按上述規則取走硬幣便算落敗，另一人則勝出。如果小華先取，他第一次要取多少枚硬幣才可確保自己勝出？

Roy plays a game with Sam as follows. There are 2 bags, with 1200 and 1800 coins respectively. They take away coins from the bags in turns according to the following rules:

- One may take away coins from only one of the bags each time. At least 1 coin must be taken away and one must not take away all the coins in the bag.
- After each round, there must be at least one bag with an even number of coins.

Whoever fails to take away coins according to the above rules is the loser while the other player is the winner. If Roy takes away coins first, how many coins should he take away in the first round in order to guarantee that he must win?

10. 如圖所示， $ABCD$  是一個邊長為 72 的正方形， $E$  為線段  $BC$  內的一點（不包括  $B$  和  $C$ ），使得  $BE$  的長度為整數。 $AE$  與  $BD$  相交於  $F$ 。若  $\triangle BEF$  的面積為整數，求此整數的最大可能值。



In the figure,  $ABCD$  is a square of side length 72.  $E$  is a point in the interior of the line segment  $BC$  (excluding  $B$  and  $C$ ) such that the length of  $BE$  is an integer.  $AE$  and  $BD$  meet at  $F$ . If the area of  $\triangle BEF$  is an integer, find the greatest possible value of this integer.

11. 現有整數  $1, 2, \dots, 50000$ 。若把它們乘起來，最後四位數字是「0000」。最少要拿走多少個整數，才可使餘下的整數之積的最後四位數字為「2005」？

Now we have the integers  $1, 2, \dots, 50000$ . If we multiply them together, the last four digits will be '0000'. At least how many integers must be removed so that the product of the remaining integers ends with '2005'?

12. 黃先生和三個女孩子嘉嘉、麗麗和敏敏玩遊戲。他有 7 張分別寫上了 1、2、3、4、5、6 和 7 的咭片。黃先生向每位女孩子派了兩張咭，並藏起了餘下的一張咭。每人原來只可以看到自己手上的咭，但後來黃先生要求每位女孩子把自己的一張咭給另一名女孩子看。於是，麗麗給嘉嘉看自己的一張咭，敏敏給麗麗看自己的一張咭，嘉嘉給敏敏看自己的一張咭。以下是他們於較後時間的對話。

黃先生問：「有人知道其他女孩子的咭上的數字嗎？」

嘉嘉說：「不知道，但我知道麗麗的數字之和是單數。」

敏敏說：「聽過嘉嘉的說話後，我仍不知道她們二人的數字是甚麼。但我知道我的數字之和比嘉嘉的數字之和大。」

麗麗說：「我知道嘉嘉和敏敏的數字了。敏敏的數字之和比我的數字之和小呢。」

假設所有女孩子都是聰明的（即當她們有足夠的資料便一定可以作出推論），求麗麗的數字之積。

Mr Wong played a game with 3 girls Karen, Lucy and Mary. There are 7 cards with numbers 1, 2, 3, 4, 5, 6 and 7 respectively. Mr Wong gave 2 cards to each girl and hid the remaining card. Originally, every girl could only see her own cards. Later, Mr Wong asked each girl to let another girl see one of her cards. As a result, Lucy let Karen see one of her cards, Mary let Lucy see one of her cards and Karen let Mary see one of her cards. Their subsequent conversations are as follows.

“Does anyone know the numbers on the cards of the other girls?” asked Mr Wong.

“No, but I know that the sum of the numbers on Lucy’s cards is odd.” replied Karen.

“After listening to Karen, I still do not know what their numbers are. But I know that the sum of my numbers is greater than that of Karen’s.” replied Mary.

“I know Karen’s and Mary’s numbers now,” said Lucy, “the sum of Mary’s numbers is smaller than that of mine.”

Assume that all girls are intelligent (i.e. they can make deductions whenever there is enough information). What is the product of the Lucy’s numbers?

## 乙部 (40 分)

### Section B (40 marks)

13. 某人用以下方法寫出一個數列：首項是「2」，然後是「1」；接著是「3」，然後有兩個「1」；之後是「4」，然後是三個「1」，如此類推。

2, 1, 3, 1, 1, 4, 1, 1, 1, ...

Someone forms a sequence as follows. The first term is '2', followed by a '1'. Next comes a '3', followed by two '1's. After that there is a '4', followed by three '1's, and so on.

2, 1, 3, 1, 1, 4, 1, 1, 1, ...

- (a) 數列的首 2005 項中，有多少項不是 1？ (4 分)  
How many of the first 2005 terms of the sequence are not 1? (4 marks)
- (b) 數列首 2005 項之和是多少？ (4 分)  
What is the sum of the first 2005 terms of the sequence? (4 marks)

如果我們把以上數列的項累積地加起來，我們可得到另一個新數列

2, 3, 6, 7, 8, 12, 13, 14, 15, ...

例如，因為原數列中首六項之和是  $2+1+3+1+1+4=12$ ，所以新數列的第六項是 12。

If we cumulatively sum up the terms of the above sequence, we can get another sequence

2, 3, 6, 7, 8, 12, 13, 14, 15, ...

For instance, since the sum of the first six terms of the original sequence is  $2+1+3+1+1+4=12$ , the sixth term of the new sequence is 12.

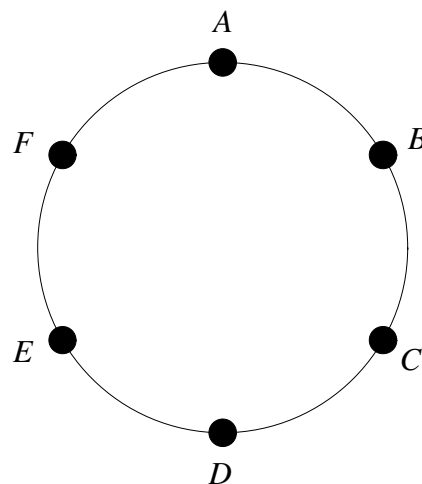
- (c) 首 2005 個正整數中，有多少個沒有在新數列中出現？ (5 分)  
How many of the first 2005 positive integers do not appear in the new sequence? (5 marks)

- (d) 志文在新數列中選了 30 個連續項，並準備了 30 根木棒，木棒的長度（以 mm 為單位）剛好等於那 30 個連續項。他發現其中三根木棒不能砌成三角形。若 30 根木棒中最長的一根的長度為  $x$  mm，求  $x$  的最大可能值。 (7 分)

Jacky picks 30 consecutive terms of the new sequence and prepares 30 wooden sticks with lengths (in mm) equal to these 30 terms. He finds that three of the wooden sticks cannot form a triangle. If the longest of the 30 wooden sticks has length  $x$  mm, find the greatest possible value of  $x$ . (7 marks)

14. 如圖所示，六個人  $A$ 、 $B$ 、 $C$ 、 $D$ 、 $E$  和  $F$  依次序圍成一個半徑為 1 的圓玩遊戲，任何相鄰的兩人之間的距離皆相同。裁判把六張分別寫上 1 至 6 的遊戲卡分派給六人。每個人只可以看到自己和他旁邊兩人的卡上的數字。每個人所得的分數就是他所看到的三張卡上的數字之積。

Six people,  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$  and  $F$ , sit around a circle of radius 1 as shown in the figure. Any two adjacent players are separated by the same distance. Six cards numbered 1 to 6 are distributed to the six people by a judge. Each person can only see the numbers on his own card and the cards of his two neighbours. The score of each person is the product of the three numbers he can see.



- (a) 若  $B$  得 24 分，求  $E$  的得分。 (3 分)  
 If the score of  $B$  is 24, find the score of  $E$ . (3 marks)
- (b)  $A$  的得分有多少個不同的可能值？ (5 分)  
 How many different possible values are there for the score of  $A$ ? (5 marks)

(c) 求六人得分總和的最大可能值。 (6分)

Find the greatest possible value of the total score of the six people. (6 marks)

(d) 後來裁判把六人分成兩組： $A$ 、 $C$ 、 $D$  一組， $B$ 、 $E$ 、 $F$  一組。每組的三位成員都可連成一個三角形，而裁判必須坐在兩個三角形的重疊位置。求裁判可坐的區域的面積。 (6分)

The judge then divides the six people into two groups:  $A$ ,  $C$ ,  $D$  form a group while  $B$ ,  $E$ ,  $F$  form another group. The three people in each group form a triangle, and the judge must sit in the region common to the two triangles. Find the area of the region in which the judge may sit. (6 marks)

**全卷完**

**END OF PAPER**