香港培正中學第三屆數學邀請賽
Pui Ching Middle School 3rd Invitational Mathematics Competition

個人賽（中三組）
Individual Event (Secondary 3)

時限：1 小時 30 分
Time allowed: 1 hour 30 minutes

參賽者須知：
Instructions to Contestants:

1. **本卷共設甲、乙兩部分，總分為** 100 **分**。
   This paper is divided into Section A and Section B. The total score is 100.

2. **除特別指明外，本卷內的所有數均為十進制**。
   Unless otherwise stated, all numbers in this paper are in decimal system.

3. **除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值**。
   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. **把所有答案填在答題紙指定的空位上。毋須呈交計算步驟**。
   Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. **不得使用計算機**。
   The use of calculators is not allowed.

6. **本卷的附圖不一定依比例繪成**。
   The diagrams in this paper are not necessarily drawn to scale.

甲部（60 分）

Section A (60 marks)

第 1 至第 4 題，每題 3 分。	Questions 1 to 4 each carries 3 marks.
第 5 至第 8 題，每題 5 分。	Questions 5 to 8 each carries 5 marks.
第 9 至第 12 題，每題 7 分。	Questions 9 to 12 each carries 7 marks.

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1. **若 p 和 q 除以 2004 時的餘數分別為 60 和 70，則 pq 除以 2004 時的餘數是多少？**

   If p and q leave remainders of 60 and 70 respectively when divided by 2004, what is the remainder when pq is divided by 2004?

2. **若 p 和 q 為質數，且 $5p+3q=91$，求 p。**

   If p and q are prime numbers such that $5p+3q=91$, find p.

3. **設 A (1, 1)、B (9, 7) 和 C (7, 1) 為平面上的三點。若 D 是 AB 上的一點，使得 $AB \perp CD$，求 CD 的長度。**

   Let A (1, 1), B (9, 7) and C (7, 1) be three points on the plane. If D is a point on AB such that $AB \perp CD$, find the length of CD.

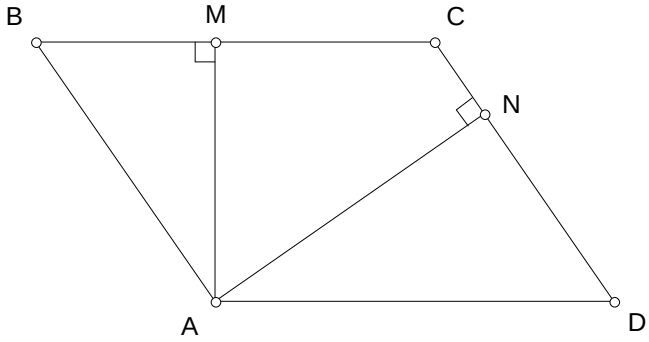
4. **乘積 11111111111×11111111111 中的數字之和是多少？**

   What is the sum of digits in the product 11111111111×11111111111?

5. **小明用木棒砌三角錐體。他有兩種分別長 10 cm 和 21 cm 的木棒。他最多可以砌出多少個不同形狀的三角錐體？（若兩個三角錐體可旋轉至對應邊長度相同，則兩個三角錐體的形狀視為相同。）**

   Peter tries to build tetrahedra with wooden sticks. He only has wooden sticks of lengths 10 cm and 21 cm. What is the largest number of tetrahedra of different shapes he can build? (Two tetrahedra are regarded to have the same shape if we can suitably rotate them to make the lengths of the corresponding edges the same.)

6. *ABCD* 為平行四邊形，*M* 和 *N* 分別是 *BC* 和 *CD* 上的點，使得 $AM \perp BC$ 和 $AN \perp CD$。若 *AB* = 13，*BM* = 5，*MC* = 9，求 *MN* 的長度。



ABCD is a parallelogram. *M* and *N* are points on *BC* and *CD* respectively such that $AM \perp BC$ and $AN \perp CD$. If *AB* = 13, *BM* = 5 and *MC* = 9, find the length of *MN*.

7. 若 $0.8+0.88+0.888+\cdots+0.\underbrace{88\ldots88}_{m\text{位}}>2004$，求 *m* 的最小值。

If $0.8+0.88+0.888+\cdots+0.\underbrace{88\ldots88}_{m\text{ digits}}>2004$, find the least value of *m*.

8. *A* 是某個半徑為 1 的圓形的圓心，*X*、*Y* 為圓周上的兩點。*Z* 是使得 *A* 為 ΔXYZ 的重心的一點。求 ΔXYZ 的面積的最大值。（註：分別把三角形的三個頂點與其對邊的中點以直線連起，則這三條直線交於一點，交點稱為三角形的重心。）

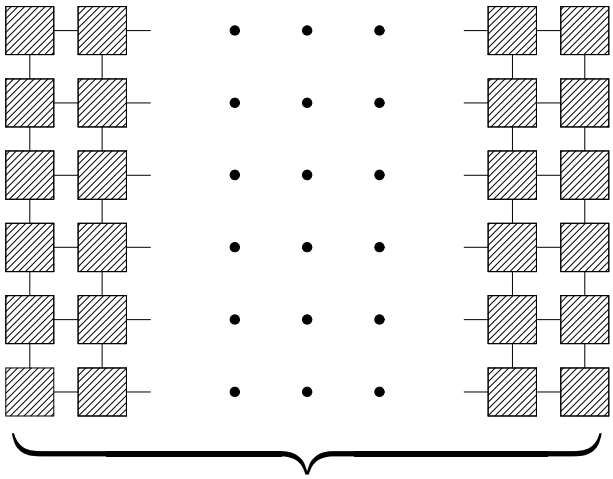
A is the centre of a circle of radius 1 and *X*, *Y* are points on the circumference. *Z* is a point such that *A* is the centroid of ΔXYZ. Find the largest possible area of ΔXYZ. (Remark: By joining each vertex of a triangle with the mid-point of its opposite side, the three straight lines meet at a point, called the **centroid** of the triangle.)

9. 一種常用的日期表示法是以「年 / 月 / 日」六位數字形式寫出日期，例如：2004 年 3 月 7 日寫成 04/03/07。由於 $04+03=07$，我們說這天是「好日子」。一般來說，若某天在以上的日期表示法中，代表「年」、「月」、「日」的三個兩位數中其中一個等於另外兩個之和，則那天稱為「好日子」。那麼，在二十一世紀中（2001 年 1 月 1 日至 2100 年 12 月 31 日），「好日子」共有多少天？

A usual way of writing dates is the 'YY/MM/DD' method of expressing a date as a six-digit number. For instance, 7th March 2004 is denoted as 04/03/07. Since $04+03=07$, we say that this is a 'good day'. In general, a day is said to be a 'good day' if, among the three two-digit numbers representing 'year', 'month' and 'day' in the above representation, one of them is equal to the sum of the other two. How many 'good days' are there in the 21st century (from 1st January 2001 to 31st December 2100)?

10. **如圖所示，384 個大小相同的正方形整齊地排成 64 列，每列 6 個，並用直線互相連起。每個正方形均被塗上紅色或綠色，使得沒有兩列的塗色完全相同。現在我們把紅色的正方形移走，那麼餘下的綠色正方形被分成若干個互不相連的部分。設最多正方形的一部分有 S 個正方形，求 S 的最小可能值。**

In the figure, 384 squares of the same size are arranged regularly in 64 columns with 6 squares in each column. The squares are connected together by straight lines. Each square is coloured red or green in a way such that no two columns have identical colouring. Now we remove the red squares, so that the remaining green squares are split into several mutually disconnected parts. Suppose that the part with the largest number of squares has S squares, find the smallest possible value of S.



64 列
64 columns

11. **一個半徑為 1 的圓形以 O 為圓心。A、B、C 是圓周上的三點。D 是使得 $ABCD$ 為平行四邊形的一點。若 A、O、D 成一直線，且 $AB:AD=1:2$，求 $ABCD$ 的周界。**

A circle centred at O has radius 1. A, B, C are three points on the circumference, and D is a point such that $ABCD$ is a parallelogram. If A, O, D are collinear and $AB:AD=1:2$, find the perimeter of $ABCD$.

12. **某國家流通的貨幣只有 \$1、\$2、\$4、\$8 和 \$16 紙幣。要付款剛好 \$23，不設找續，共有多少種方法？**

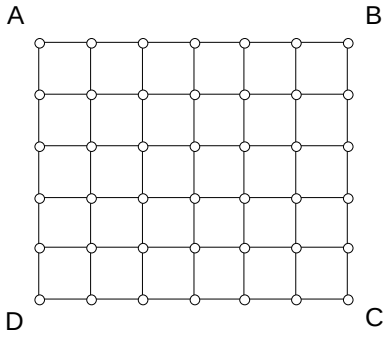
The currency in a country consists of only \$1, \$2, \$4, \$8 and \$16 notes. How many ways are there to pay exactly \$23 if no change is allowed?

乙部（40 分）

Section B (40 marks)

13. *ABCD* 是一個長 *p*、闊 *q* 的長方形，並被分成 *pq* 個邊長為 1 的小正方形。這些小正方形的頂點稱為「格點」。圖中顯示了 $p = 5$、$q = 6$ 時的情況。當 *p* 和 *q* 的值有所不同時，*ABCD* 的形狀和大小亦會隨之而改變。

ABCD is a rectangle of length *p* and width *q*. It is divided into *pq* small squares of side length 1. Vertices of these small squares are said to be 'lattice points'. The figure shows the case with $p = 5$ and $q = 6$. The shape and size of *ABCD* vary as the values of *p* and *q* change.



(a) 當 $p = 99$，$q = 2004$ 時，圖中會有多少個格點？ （3 分）

If $p = 99$ and $q = 2004$, how many lattice points will there be in the figure? (3 marks)

(b) 設 $p = 48$，$q = 72$。直線 *AC* 上有多少個格點（包括 *A* 和 *C*）？ （4 分）

Suppose $p = 48$ and $q = 72$. How many lattices points are there on the straight line *AC* (including *A* and *C*)? (4 marks)

(c) 設 $p = 3$，$q = 4$。現要選取四個格點，以組成長方形（包括正方形）。那麼，共有多少種不同的選法？ （6 分）

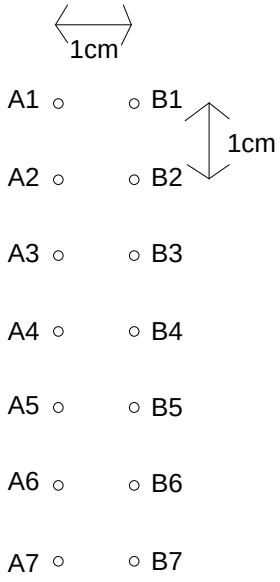
Suppose $p = 3$ and $q = 4$. Now we want to form rectangles (including squares) by picking four lattice points. How many different choices are there? (6 marks)

(d) 小明沿直線 *AC* 把長方形剪開。他發現剛好有 2004 個小正方形被分割成兩部分。把 *ABCD* 的周界記為 *S*。已知 *S* 有多個不同的可能值。在這些可能值中，求最接近 5000 的一個。 （7 分）

Peter cuts the rectangle along the straight line *AC*. He finds that exactly 2004 of the small squares are split into two parts. Denote the perimeter of *ABCD* by *S*. It is known that *S* has many different possible values. Among these possible values, find the one that is closest to 5000. (7 marks)

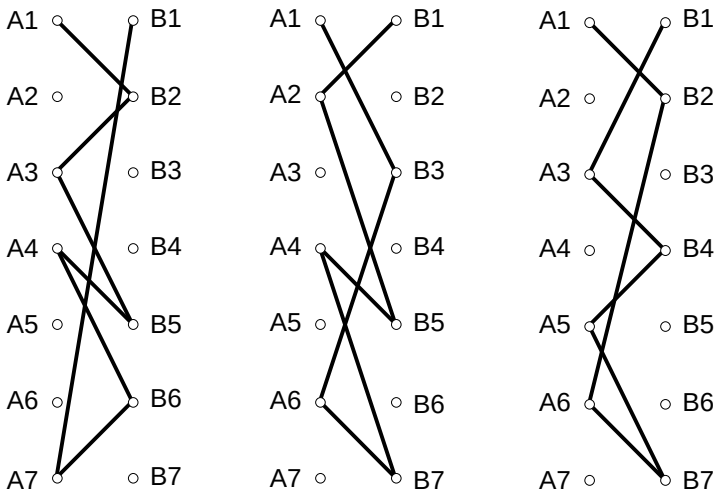
14. **小月要為一隻鞋穿鞋帶。鞋上有兩列、每列七個鞋帶孔，這些鞋帶孔整齊地在平面上排成一個長方形，相鄰的鞋帶孔相隔 1 厘米（見圖一）。小月穿鞋帶時，每次均會由 A1 孔開始穿，穿到 B 列的某一個孔，再穿回 A 列的另一個孔，如此類推，一直穿回 B1 孔為止。此外，為了節省時間起見，除第一行（A1 和 B1 孔）外，鞋帶只會同時穿過同一行的兩個孔中的剛好一個（即不會同時穿過 A2 和 B2，不會同時穿過 A3 和 B3，如此類推）。鞋帶只可以穿過每個孔最多一次。圖二所示的是一些符合上述規則的穿鞋帶方法。**

Mary is tying her shoelace. There are 2 columns of holes, each with 7 holes on it. The holes are evenly distributed on a plane in a rectangular shape, with each of the holes at a distance of 1 cm from its neighbours (see Figure 1). When Mary ties the shoelace, she always ties the shoelace through hole A1 first, then to a hole on column B, and then back to a hole on column A and so on, until she returns to hole B1. In order to save time, Mary will only tie the shoelace through exactly one of the two holes on the same row except for the first row (holes A1 and B1), i.e. the shoelace will not go through both A2 and B2, nor both A3 and B3, etc. The shoelace can pass through each hole at most once. Figure 2 shows some possible configurations of the shoelace under the above rules.



圖一

Figure 1



圖二

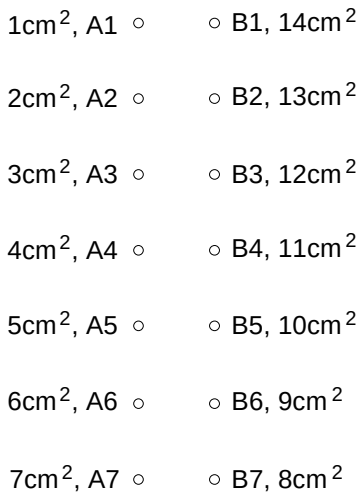
Figure 2

(a) **小月有多少種不同的方法穿鞋帶？（只要穿孔的次序相同，兩種穿鞋帶的方法便視為相同。）** **（4 分）**

In how many different ways can Mary tie the shoelace? (Two ways are regarded to be the same if the orders by which the shoelace passes through the holes are the same in both ways.) (4 marks)

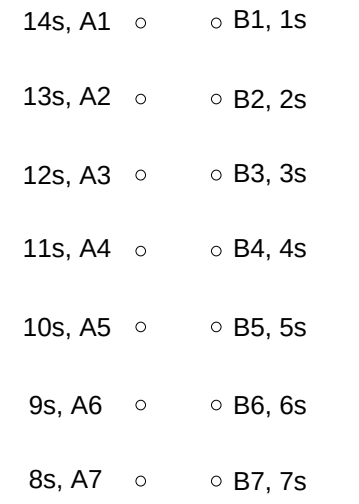
(b) **圖中所示為各鞋帶孔的面積。若鞋帶所穿過的孔的面積之和是 S cm^2，那麼 S 除以 15 時的餘數有多少個不同的可能值？** **（6 分）**

The figure shows the areas of the holes. Suppose that the sum of the areas of the holes which the shoelace passes through is S cm^2. How many different remainders are possible when S is divided by 15? (6 marks)



(c) **圖中所示為穿過各鞋帶孔所需的時間。那麼，小月穿鞋帶最少需要多少秒？** **（4 分）**

The figure shows the time needed to tie the shoelace through each of the holes. What is the minimum amount of time (in seconds) that Mary needs to tie the shoelace? (4 marks)



(d) **求由 A1 孔到 B1 孔的鞋帶的長度的最小值。** **（6 分）**

Find the minimum length of the shoelace from hole A1 to hole B1. (6 marks)

全卷完

END OF PAPER

個人賽（中三組）答案

Individual Event (Secondary 3) Answers

1. 192

2. 17

3. $\frac{18}{5}$

4. 85

13 (a) 200500

13 (b) 25

13 (c) 74

13 (d) 5010

5. 5

6. $\frac{180}{13}$

7. 2255

8. $\frac{3}{2}$

14 (a) 720

14 (b) 10

14 (c) 51

14 (d) $5\sqrt{5}+2\sqrt{2}$

9. 730

10. 6

11. $6\sqrt{3}-6$

12. 74