香港培正中學第一屆數學邀請賽

Pui Ching Middle School 1st Invitational Mathematics Competition

團體賽（高級組）

Group Event (Senior Section)

時限：45 分鐘

Time allowed: 45 minutes

參賽者須知：

Instructions to Contestants:

1. **本卷共設** 20 **題，總分為** 100 **分。**

   There are 20 questions in this paper and the total score is 100.

2. **除特別指明外，本卷內的所有數均為十進制。**

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. **除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。**

   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. **所有答案填在答題紙指定的空位上。毋須呈交計算步驟。**

   Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. **不得使用計算機。**

   The use of calculators is not allowed.

6. **本卷的附圖不一定依比例繪成。**

   The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 5 題，每題 2 分。

Questions 1 to 5 each carries 2 marks.

1. **若九位數 2002k0302 可被 3 整除，求 k 所有可能值之和。**

   If the 9-digit number 2002k0302 is divisible by 3, find the sum of all possible values of k.

2. **求最小的正整數 n，使得 $80-n$ 和 $80+n$ 均為質數。**

   Find the smallest positive integer n for which both $80-n$ and $80+n$ are prime.

3. **一間文具店有鉛筆出售：每支鉛筆售 1 元，5 支裝售 4 元，17 支裝售 12 元。如果李先生需要 2002 支鉛筆，並付了 n 元買鉛筆，那麼 n 的最小值是甚麼？（注意他買的鉛筆可能比他需要的多。）**

   A shop sells pencils. Each pencil costs 1 dollar, a bulk package of 5 pencils costs 4 dollars and a bulk package of 17 pencils costs 12 dollars. If Mr Lee needs 2002 pencils and he pays n dollars to buy pencils, what is the smallest value of n? (Note that he may buy more pencils than he needs.)

4. **某年六月的平均氣溫為 29 度。若該年六月一日至十日的平均氣溫為 25 度，則該年六月十一日至三十日的平均氣溫是多少度？**

   The average temperature in June of a certain year was 29 degrees. If the average temperature from 1st to 10th June that year was 25 degrees, what was the average temperature from 11th to 30th June that year in degrees?

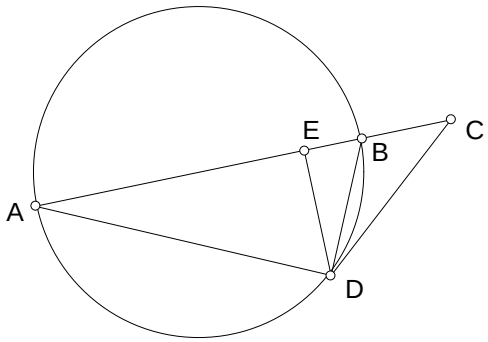
5. **某次數學測驗共有 20 題。每題答對可得 5 分，不答得 0 分，答錯扣 2 分。若小麗在測驗中的得分為質數，問她最多答對了幾題？**

   In a mathematics test there were 20 questions. 5 marks were awarded for each correct answer, 0 mark would be given if a question is left unanswered, and 2 marks were deducted for each wrong answer. If the score of Lily in the test is a prime number, what is the largest number of questions she answered correctly?

第 6 至第 10 題，每題 4 分。

Questions 6 to 10 each carry 4 marks.

6. **如圖所示，*AB* 為圓的直徑，*C* 為 *AB* 延線上的一點，*CD* 切圓於 *D*，且 *E* 為 *D* 到 *AB* 的垂足。若 $AD = 6$，且 $\angle CDB = 30°$，求 ΔDEB 的面積。**



In the figure, *AB* is a diameter of the circle and *C* is a point on *AB* produced. *CD* is tangent to the circle at *D* and *E* is the foot of the perpendicular from *D* to *AB*. If $AD = 6$ and $\angle CDB = 30°$, find the area of ΔDEB.

7. **彼得和瑪莉各自選了一個不超過 10000 的正整數。已知彼得所選的數是 3 的倍數，瑪莉所選的數是 5 的倍數。問兩人所選的數相同的概率是多少？**

Peter and Mary each chose a positive integer not exceeding 10000. It is known that Peter's number is divisible by 3 while Mary's number is divisible by 5. What is the probability that the two numbers are the same?

8. **對所有正整數 n，定義 $1+2+3+...+n$ 為第 n 個三角形數。有多少個三角形數不是合成數？**

For all natural numbers n, define the n-th triangular number by $1+2+3+...+n$. How many triangular numbers are not composite?

9. **三根鐵棒 *A*、*B* 和 *C* 垂直地插進盛了水的水池中。原先 *A*、*B*、*C* 露出水面部份的長度之比為 $1:2:4$。下雨後，水深是原來的兩倍。那時，*A*、*B*、*C* 露出水面部份的長度之比為 $1:3:7$。若 *A* 的長度為 20 米，且 *C* 的長度為 *x* 米，求 *x*。**

Three iron bars, *A*, *B* and *C*, were inserted vertically into a water tank with water. Initially, the ratio of the lengths of *A*, *B* and *C* above water was $1:2:4$. After raining, the depth of water was doubled. At that time, the ratio of the lengths of *A*, *B* and *C* above water became $1:3:7$. If the length of *A* is 20 metres and the length of *C* is *x* metres, find *x*.

10. **張先生到市場買雞。已知每隻公雞、母雞和小雞的售價分別是 24 元、16 元和 1 元。他用了 400 元買了 100 隻雞。他買了多少隻母雞？**

Mr. Cheung bought chickens from a market. It is known that a cock, a hen and a chick cost $24, $16 and $1 respectively. He bought 100 chickens with $400. How many hens did he buy?

第 11 至第 15 題，每題 6 分。

Questions 11 to 15 each carries 6 marks.

11. **某人到超級市場購物，付款 500 元。收銀員找贖時誤把 100 元紙幣當成 50 元，50 元紙幣當成 20 元，20 元紙幣當成 100 元。已知收銀員共找回紙幣六張，且超級市場因她出錯而損失了 110 元。問該人所選購的貨品總值多少元？**

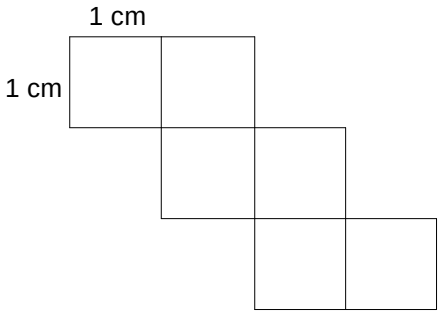
    A man went shopping in a supermarket and paid $500. When returning the change, the cashier misregarded the $100 notes as $50 notes, $50 notes as $20 notes and $20 notes as $100 notes. Given that 6 notes have been returned as change and the supermarket lost $110 as a result of the cashier' s fault, how many dollars were the items bought by the man worth?

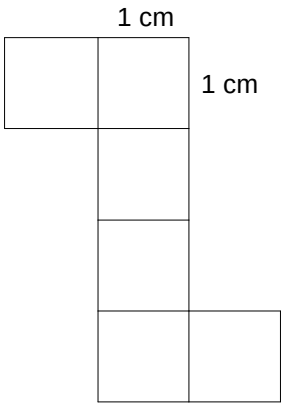
12. **對於正整數 *n*，我們定義 *n* 的「左面」為 *n* 除去其最右邊的數字所構成的數；相似地，定義 *n* 的「右面」為 *n* 除去其最左邊的數字所構成的數。例如，12345 的「左面」和「右面」分別為 1234 和 2345。如果 *n* 的「左面」和「右面」之差是 9 的倍數，則稱 *n* 為「好數」，否則 *n* 就不是「好數」。那麼 10000 和 100000 之間共有多少個「好數」？**

    For positive integers *n*, we define the 'left' of *n* as the number formed by removing its rightmost digit. Similarly, we define the 'right' of *n* as the number formed by removing its leftmost digit. For example, the 'left' and 'right' of 12345 are 1234 and 2345 respectively. If the difference between the 'left' and the 'right' of *n* is a multiple of 9, then we say that *n* is a 'good number'; otherwise *n* is not a 'good number'. How many 'good numbers' are there between 10000 and 100000?

13. **附圖是一些可以摺成一個邊長為 1 厘米的立方體的紙樣。現要摺一個長 3 厘米，闊 4 厘米，高 5 厘米的長方體。如果所需要的紙樣的周界最短是 n 厘米，求 n。**

The figures below show some nets which can be folded to form a cube with side length 1 cm. Now a rectangular block which is 3 cm long, 4 cm wide and 5 cm high is to be folded. If the minimum perimeter of the net needed is n cm, find n.



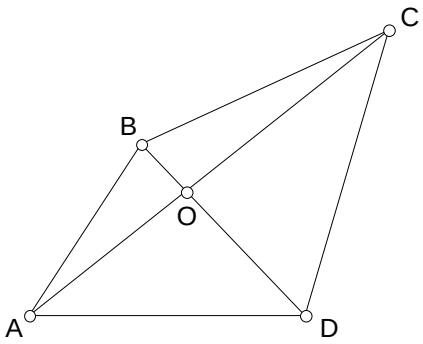


14. **一張尺寸為 11×11 的正方形白色氈子被分成 121 個尺寸為 1×1 的方格。氈子正中間的白色方格隨後被塗上黑色。那麼包括這個黑色方格的長方形（包括正方形）共有多少個？**

A square white mat of size 11×11 is divided into 121 squares of size 1×1. The square in the middle of the mat is then painted black. How many rectangles (INCLUDING squares) contain this black square?

15. **我們以 $[XYZ]$ 來表示 ΔXYZ 的面積。圖中，$ABCD$ 為四邊形，AC 交 BD 於 O。若 $AO=4$、$CO=5$、$DO=3$、$AD=6$，且 $BD=CD$，求 $\dfrac{[AOB]}{[COD]}$ 的值。**

We denote the area of ΔXYZ by $[XYZ]$. In the figure, $ABCD$ is a quadrilateral; AC and BD meet at O. If $AO = 4$, $CO = 5$, $DO = 3$, $AD = 6$ and $BD = CD$, find the value of $\dfrac{[AOB]}{[COD]}$.



第 16 至第 20 題，每題 8 分。

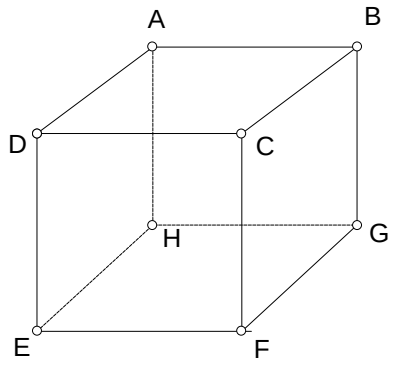
Questions 16 to 20 each carries 8 marks.

16. **有一個壞了大部份按鍵的計算機，只剩下兩個數字鍵「5」和「8」和兩個功能鍵「+」（加號）和「=」（等號）可正常操作。現在它的螢幕上顯示著「0」。計算機的設計不容許連續兩次按功能鍵。如果我們希望通過運算令螢幕顯示「2002」，最少需要按數字鍵（不包括功能鍵）多少次？（註：輸入整數時每一個數字皆需按一次鍵，例如輸入「558」和「85885」時分別需要按 3 次和 5 次鍵。）**

There is a broken calculator with only two number keys '5' and '8' and two function keys '+' (plus) and '=' (equals) still working well. '0' is shown on its screen now. The design of the calculator does not allow two consecutive presses of function keys. If we want to show '2002' on its screen through calculations, what is the least number of presses of number keys (NOT including function keys) we need? (Note: When inputting an integer, each digit needs a press to be input. For example, inputting '558' and '85885' need 3 and 5 presses of number keys respectively.)

17. **如圖所示，*ABCDEFGH* 為正方體。若四面體 *ACEG* 的體積為 243，求 *ABCD* 的面積。**

In the figure, *ABCDEFGH* is a cube. If the volume of tetrahedron *ACEG* is 243, find the area of *ABCD*.



18. **若 $3^{\frac{1}{3}}\times 9^{\frac{1}{9}}\times 27^{\frac{1}{27}}\times 81^{\frac{1}{81}}\times\cdots$ 無窮項之積等於 3^k，求 k。**

If the product of the infinitely many terms $3^{\frac{1}{3}}\times 9^{\frac{1}{9}}\times 27^{\frac{1}{27}}\times 81^{\frac{1}{81}}\times\cdots$ is equal to 3^k, find k.

19. **設 $[x]$ 為不超過 x 的最大整數，例如：$[1.99]=1$，$[2.1]=2$，$[7]=7$。**

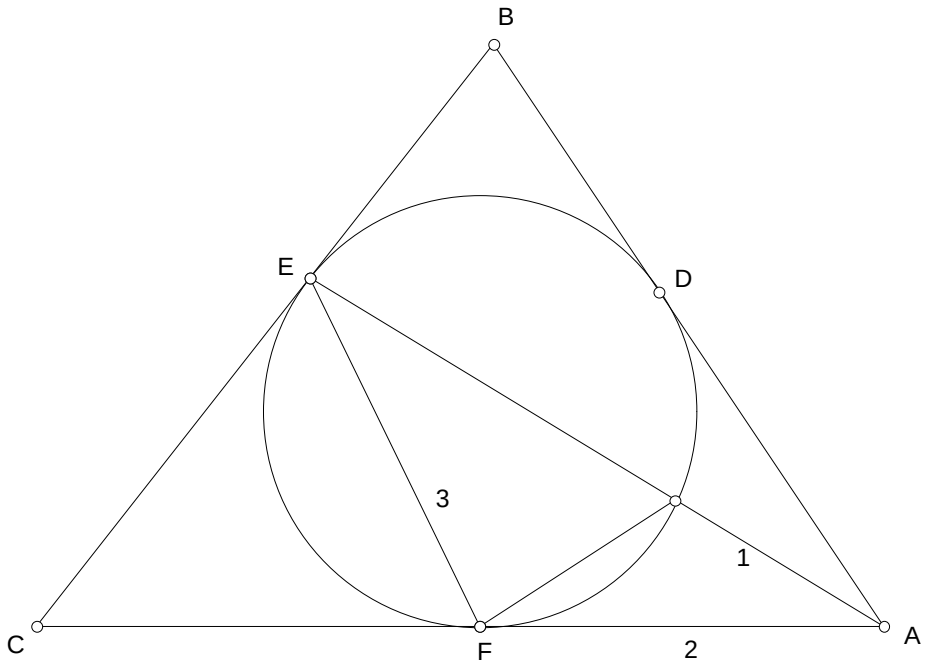
求 $\left[30\left(\frac{1}{1\sqrt{4}+4\sqrt{1}}+\frac{1}{4\sqrt{7}+7\sqrt{4}}+\frac{1}{7\sqrt{10}+10\sqrt{7}}+\cdots+\frac{1}{1999\sqrt{2002}+2002\sqrt{1999}}\right)\right]$ 的值。

Let $[x]$ be the greatest integer not exceeding x. For example, $[1.99]=1$, $[2.1]=2$, $[7]=7$.

Find the value of $\left[30\left(\frac{1}{1\sqrt{4}+4\sqrt{1}}+\frac{1}{4\sqrt{7}+7\sqrt{4}}+\frac{1}{7\sqrt{10}+10\sqrt{7}}+\cdots+\frac{1}{1999\sqrt{2002}+2002\sqrt{1999}}\right)\right]$.

20. **如圖所示，圓形 $DEFG$ 為三角形 ABC 的內切圓，其中 AB、BC 及 CA 分別切圓於 D、E 及 F，且 AGE 是直線。若 $AG=1$，$AF=2$，$EF=3$，求三角形 ABC 的周界。**

In the figure, circle $DEFG$ is inscribed in triangle ABC, where AB, BC and CA touch the circle at D, E and F respectively and AGE is a straight line. If $AG=1$, $AF=2$ and $EF=3$, find the perimeter of triangle ABC.



全卷完

END OF PAPER

團體賽（高級組）答案

Group Event (Senior Section) Answers

1. 18
2. 9
3. 1415
4. 31
5. 17
6. $\frac{3}{2}\sqrt{3}$
7. $\frac{111}{1111000}$
8. 2
9. 60
10. 20
11. 240
12. 10000
13. 50
14. 1296
15. $\frac{2}{5}$
16. 14
17. 81
18. $\frac{3}{4}$
19. 9
20. 20