

香港培正中學第一屆數學邀請賽

Pui Ching Middle School 1st Invitational Mathematics Competition

個人賽 (中三組)

Individual Event (Secondary 3)

時限：1 小時 30 分

Time allowed: 1 hour 30 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 5 題，每題 2 分。

Questions 1 to 5 each carries 2 marks.

1. 定義 $a*b = 3a + 4b + 5$ 。若 $x*2 = 2002$ ，求 x 。

Define $a*b = 3a + 4b + 5$. If $x*2 = 2002$, find x .

2. 一筆金錢，由甲、乙、丙、丁四人分享。甲、乙共分得總數的 40%，乙、丙共分得總數的 50%，甲、丙共分得總數的 60%，且丙分得 4200 元。問丁分得多少元？

A sum of money was shared among A , B , C and D . A and B together got 40% of the total amount; B and C together got 50% of the total amount; A and C together got 60% of the total amount. It is known that C got \$4200. How many dollars did D get?

3. 求 $7 \times 9 \times 11 \times \cdots \times 1999 \times 2001 \times 2003$ 的個位數字。

Find the unit digit of $7 \times 9 \times 11 \times \cdots \times 1999 \times 2001 \times 2003$.

4. 已知當 $\frac{1}{19}$ 寫成循環小數時是 $0.\dot{0}5263157894736842\dot{1}$ 。如果將 $\frac{10}{19}$ 寫成循環小數，小數點後第 2002 個位的數字是甚麼？

Given that when $\frac{1}{19}$ is written as a recurring decimal, it is $0.\dot{0}5263157894736842\dot{1}$. If $\frac{10}{19}$ is written as a recurring decimal, what is the 2002nd digit after the decimal point?

5. 一個足球聯賽共有 20 隊，採取單循環賽制（即任何兩隊皆會對賽剛好 1 次）。在每場比賽中，若兩隊賽和則雙方各得 1 分，否則勝方得 2 分，負方得 0 分。當所有賽事完成後，分數由高至低排，問第 18 名的隊伍最少有多少分？

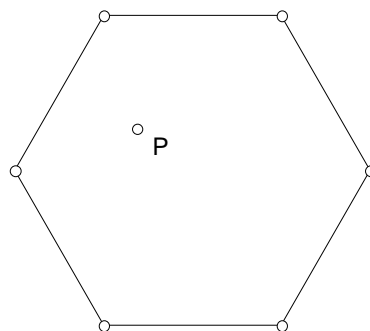
There are 20 teams in a football league, where every team will play exactly 1 match against each other team. In each match, if the game is a draw, 1 point is awarded to each team. Otherwise, 2 marks will be awarded to the winner and 0 mark will be awarded to the loser. After all matches are played, the teams are ranked according to their marks, from highest to lowest. What is the minimum number of marks the team at the 18th position may have?

第 6 至第 10 題，每題 4 分。

Questions 6 to 10 each carries 4 marks.

6. 如圖所示，一個正六邊形內有一點 P 。已知正六邊形每條邊的邊長為 $2\sqrt{3}$ 。若以 d_1, d_2, \dots, d_6 表示 P 點與六邊形六條邊的垂直距離，求 $d_1 + d_2 + \dots + d_6$ 的值。

As shown in the figure, P is a point in a regular hexagon. Given that each side of the regular hexagon is $2\sqrt{3}$. If we denote the perpendicular distances of P from the six sides of the hexagon by d_1, d_2, \dots, d_6 , find the value of $d_1 + d_2 + \dots + d_6$.



7. 求 9991 的最大質因數。

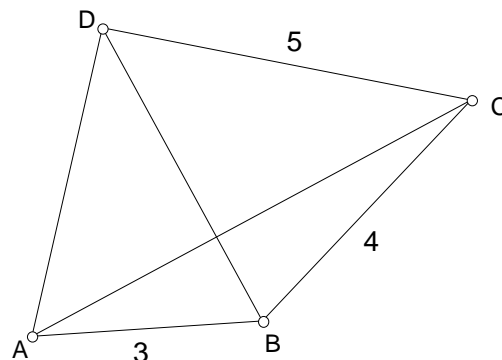
Find the largest prime factor of 9991.

8. 若 $0 \leq x \leq 90$ ， $0 \leq y \leq 90$ ，且 $(1 + \sin x^\circ)^{2002} = \cos y^\circ$ ，求 $x + y$ 的值。

If $0 \leq x \leq 90$ ， $0 \leq y \leq 90$ and $(1 + \sin x^\circ)^{2002} = \cos y^\circ$ ，find the value of $x + y$.

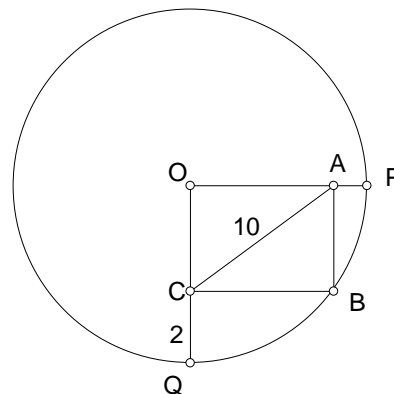
9. 圖中，四邊形 $ABCD$ 的對角線 AC 及 BD 互相垂直。若 $AB = 3$ ， $BC = 4$ ， $CD = 5$ ，求 AD 的長度。

In the figure, the diagonals AC and BD of quadrilateral $ABCD$ are perpendicular to each other. If $AB = 3$ ， $BC = 4$ ， $CD = 5$ ，find the length of AD .



10. 如圖所示， O 為圓形的圓心， B 為圓周上的一點，且 $OABC$ 為長方形。 OA 與 OC 延長後分別交圓於 P 及 Q 。若 $AC = 10$ ，且 $CQ = 2$ ，求 AP 。

In the figure, O is the centre of the circle, B is a point on the circumference and $OABC$ is a rectangle. OA and OC are produced to meet the circle at P and Q respectively. If $AC = 10$ and $CQ = 2$ ，find AP .



第 11 至第 15 題，每題 6 分。

Questions 11 to 15 each carries 6 marks.

11. 求 $\frac{\tan 1^\circ}{1 + \tan 1^\circ} + \frac{\tan 2^\circ}{1 + \tan 2^\circ} + \frac{\tan 3^\circ}{1 + \tan 3^\circ} + \dots + \frac{\tan 89^\circ}{1 + \tan 89^\circ}$ 的值。

Find the value of $\frac{\tan 1^\circ}{1 + \tan 1^\circ} + \frac{\tan 2^\circ}{1 + \tan 2^\circ} + \frac{\tan 3^\circ}{1 + \tan 3^\circ} + \dots + \frac{\tan 89^\circ}{1 + \tan 89^\circ}$.

12. 計算 $1^2 - 2^2 - 3^2 + 4^2 + 5^2 - 6^2 - 7^2 + 8^2 + \dots + 1997^2 - 1998^2 - 1999^2 + 2000^2 + 2001^2 - 2002^2$

Compute $1^2 - 2^2 - 3^2 + 4^2 + 5^2 - 6^2 - 7^2 + 8^2 + \dots + 1997^2 - 1998^2 - 1999^2 + 2000^2 + 2001^2 - 2002^2$.

13. 一所飲品公司正推行一項以印花換領飲品的計劃。此飲品乃非賣品，但每 11 個印花可換領 1 瓶飲品。而且，每瓶飲品上皆有 1 個印花以作日後換領之用。如果某人希望換領 20020302 瓶飲品，那麼他最少需要多少個印花？

A beverage company is promoting a scheme of drink redemption by stamps. The drinks are not for sale, but every 11 stamps can redeem a bottle of drink. Moreover, there is a stamp on each bottle of drink for further redemption. If someone wants to redeem 20020302 bottles of drink, how many stamps does he need?

14. 在一個正方形內畫 2002 點，並把其中一些點用線段連起，再沿這些線段把正方形切割成一些三角形，而這些三角形的頂點均為正方形的頂點或該 2002 點中的一些點。問最多可得到多少個三角形？

2002 points are drawn inside a square. Some of these points are connected by line segments and the square is cut along these line segments into a number of triangles, each of which has vertices among either the 2002 points or the vertices of the square. What is the largest number of triangles obtained?

15. 設 x 、 y 、 z 為滿足以下方程組的實數。求 $2x + 3y + 5z$ 的最小值。

Let x , y , z be real numbers satisfying the following system of equations. Find the minimum value of $2x + 3y + 5z$.

$$\begin{cases} (x+y)(x+y+z) = 165 \\ (y+z)(x+y+z) = 150 \\ (z+x)(x+y+z) = 135 \end{cases}$$

第 16 至第 20 題，每題 8 分。

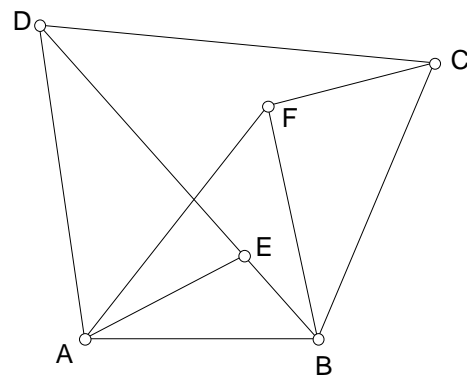
Questions 16 to 20 each carries 8 marks.

16. 設 x 和 y 為正整數。當 x 除以 y 時，商為 4321，餘數為 1234。當 x 除以 $2y$ 時，餘數的最小可能值是多少？

Let x and y be positive integers. When x is divided by y , the quotient is 4321 and the remainder is 1234. What is the smallest possible remainder when x is divided by $2y$?

17. 如圖所示， E 為四邊形 $ABCD$ 對角線 BD 上的一點。若 $AB = 4$ ， $BC = 8$ ， $CD = 12$ ， $DB = 10$ ， $BF = 5$ ， $FA = 6$ ， $AE = 3$ ，且 $EB = 2.5$ ，求 CF 的長度。

In the figure, E is a point on diagonal BD of quadrilateral $ABCD$. If $AB = 4$, $BC = 8$, $CD = 12$, $DB = 10$, $BF = 5$, $FA = 6$, $AE = 3$ and $EB = 2.5$, find the length of CF .



18. 某個會有 2002 名會員。除該會的創會會員外，每人均需得到剛好一名已入會的會員推薦方能入會。每名會員獲發一張會員證。基於行政上的理由，每名非創會會員的會員證均需與推薦人的會員證同色，而不同創會會員之會員證的顏色不同。若一名會員曾經推薦新會員入會，則被推薦的人數必定為 12、30 或 42。此外，沒有會員於入會後退會。問該會的會員證至少有多少種不同的顏色？

A club has 2002 members. Except for its founders, each person must be nominated by exactly one existing member before he can join the club. Every member is issued a membership card. For administrative purpose, the membership card of each non-founding member must be of the same colour as that of one who nominated him, and the membership cards of different founders are of different colours. If a member of the club has ever nominated new members, the number of nominees must be 12, 30 or 42. Also, no member ever drops out after joining the club. What is the minimum number of different colours of the membership cards?

19. 董先生參加某國家的總統選舉，得票率（準確至小數點後一個位）為 66.6%。問董先生最少得到多少票？

Mr Tung joined the presidential election of a country and obtained 66.6% of the total votes, correct to 1 decimal place. What is the minimum number of votes Mr Tung obtained?

20. 有一個正方體，現把每個頂點塗上紅色或藍色，使得紅色頂點比藍色頂點多。問有多少種不同的塗色方法？（若兩個正方體不論如何旋轉均無法達致對應頂點顏色相同，則我們說這兩個正方體的塗色方法不同，否則我們說這兩個正方體的塗色方法相同。）

There is a cube and each of its vertices is coloured red or blue, such that there are more red vertices than blue vertices. How many different ways of colourings are there? (Two cubes are said to have different colourings if it is impossible to make the colours of their corresponding vertices the same upon any rotation. Otherwise we say that they have the same colouring.)

全卷完

END OF PAPER

個人賽 (中三組) 答案

Individual Event (Secondary 3) Answers

1. 663

11. 44.5

2. 3000

12. -2003

3. 5

13. 200203021

4. 3

14. 4006

5. 2

15. -48

6. 18

16. 2469

7. 103

17. 6

8. 0

18. 4

9. $3\sqrt{2}$

19. 191

10. 4

20. 8