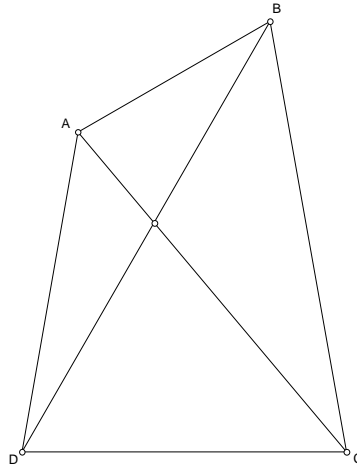


PROBLEM.

Refer to the figure, $ABCD$ is a quadrilateral such that $\angle ADB = 20^\circ$, $\angle BDC = 60^\circ$, $\angle BCA = 30^\circ$, $\angle ACD = 50^\circ$. Find $\angle ABC$.



SOLUTION.

Take point E on BC such that $\angle EDC = 20^\circ$.

Note that $\angle DEC = \angle DCE (= 80^\circ)$ and follows that $DC = DE$.

On the other hand, $DC = DA$ because $\angle DAC = \angle DCA (= 50^\circ)$.

So, $DE = DA$.

We note also that $\angle ADE = 60^\circ$, which implies $\triangle ADE$ is equilateral.

Now, consider $\triangle BDE$. We see $\angle EDB = 40^\circ$ and $\angle EBD = 40^\circ$, hence

$EB = ED = EA$.

In the isosceles $\triangle EAB$, $\angle AEB = 40^\circ$. So $\angle EAB = \angle EBA = 70^\circ$.

Hence, $\angle ABC = 70^\circ$.

