

常用積分表

1. $\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$
2. $\int (f(x) - g(x))dx = \int f(x)dx - \int g(x)dx$
3. $\int f(x)dg(x) = f(x)g(x) - \int g(x)df(x)$
4. $\int a^x dx = \frac{a^x}{\ln a} + C, a \neq 1, a > 0$
5. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
6. $\int \frac{1}{x} dx = \ln|x| + C$
7. $\int e^x dx = e^x + C$
8. $\int \sin x dx = -\cos x + C$
9. $\int \cos x dx = \sin x + C$
10. $\int \sec^2 x dx = \tan x + C$
11. $\int \csc^2 x dx = -\cot x + C$
12. $\int \sec x \tan x dx = \sec x + C$
13. $\int \csc x \cot x dx = -\csc x + C$
14. $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C, a \neq 0, n \neq -1$
15. $\int (ax + b)^{-1} dx = \frac{1}{a} \ln|ax + b| + C, a \neq 0$
16. $\int x(ax + b)^n dx = \frac{(ax + b)^{n+1}}{a^2} \left(\frac{ax + b}{n+2} - \frac{b}{n+1} \right) + C, a \neq 0, n \neq -1, -2$
17. $\int x(ax + b)^{-1} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax + b| + C, a \neq 0$
18. $\int x(ax + b)^{-2} dx = \frac{1}{a^2} \left(\ln|ax + b| + \frac{b}{ax + b} \right) + C, a \neq 0$

19. $\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln \left| \frac{x}{ax+b} \right| + C, \quad b \neq 0$
20. $\int (\sqrt{ax+b})^n dx = \frac{2(\sqrt{ax+b})^{n+2}}{a(n+2)} + C, \quad a \neq 0, \quad n \neq -2$
21. $\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}}$
22. $\int \frac{dx}{x\sqrt{ax+b}} = \frac{2}{\sqrt{|b|}} \tan^{-1} \sqrt{\frac{ax+b}{|b|}} + C, \quad b < 0$
23. $\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C, \quad b > 0$
24. $\int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}} + C$
25. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax+b}} + C, \quad b \neq 0$
26. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$
27. $\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$
28. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C, \quad a \neq 0$
29. $\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{2a^2} \int \frac{dx}{a^2-x^2}, \quad a \neq 0$
30. $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln(x + \sqrt{a^2+x^2}) + C$
31. $\int \sqrt{a^2+x^2} dx = \frac{x}{2} \sqrt{a^2+x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2+x^2}) + C$
32. $\int x^2 \sqrt{a^2+x^2} dx = \frac{x}{8} (a^2+2x^2) \sqrt{a^2+x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2+x^2}) + C$
33. $\int \frac{\sqrt{a^2+x^2}}{x} dx = \sqrt{a^2+x^2} - a \ln \left| \frac{a + \sqrt{a^2+x^2}}{x} \right| + C$
34. $\int \frac{\sqrt{a^2+x^2}}{x^2} dx = \ln(x + \sqrt{a^2+x^2}) - \frac{\sqrt{a^2+x^2}}{x} + C$
35. $\int \frac{x^2}{\sqrt{a^2+x^2}} dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2+x^2}) + \frac{x\sqrt{a^2+x^2}}{2} + C$

36. $\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2+x^2}}{x} \right| + C$
37. $\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C, a \neq 0$
38. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C, a \neq 0$
39. $\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C, a \neq 0$
40. $\int x^2 \sqrt{a^2-x^2} dx = \frac{a^4}{8} \sin^{-1} \frac{x}{a} - \frac{1}{8} x \sqrt{a^2-x^2} (a^2-2x^2) + C, a \neq 0$
41. $\int \frac{\sqrt{a^2-x^2}}{x} dx = \sqrt{a^2-x^2} - a \ln \left| \frac{a+\sqrt{a^2-x^2}}{x} \right| + C$
42. $\int \frac{\sqrt{a^2-x^2}}{x^2} dx = -\sin^{-1} \frac{x}{a} - \frac{\sqrt{a^2-x^2}}{x} + C, a \neq 0$
43. $\int \frac{x^2}{\sqrt{a^2-x^2}} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2-x^2} + C, a \neq 0$
44. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2-x^2}}{x} \right| + C, a \neq 0$
45. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C, a \neq 0$
46. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln \left| x + \sqrt{x^2-a^2} \right| + C$
47. $\int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2-a^2} \right| + C$
48. $\int (\sqrt{x^2-a^2})^n dx = \frac{x(\sqrt{x^2-a^2})^n}{n+1} - \frac{na^2}{n+1} \int (\sqrt{x^2-a^2})^{n-2} dx, n \neq -1$
49. $\int \frac{dx}{(\sqrt{x^2-a^2})^n} = \frac{x(\sqrt{x^2-a^2})^{2-n}}{(2-n)a^2} + \frac{n-3}{(2-n)a^2} \int \frac{dx}{(\sqrt{x^2-a^2})^{n-2}}, n \neq 2$
50. $\int x(\sqrt{x^2-a^2})^n dx = \frac{(\sqrt{x^2-a^2})^{n+2}}{n+2} + C, n \neq -2$

$$51. \int x^2 \sqrt{x^2 - a^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$52. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C, \quad a \neq 0$$

$$53. \int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + \frac{x}{2} \sqrt{x^2 - a^2} + C$$

$$54. \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = \ln|x + \sqrt{x^2 - a^2}| - \frac{\sqrt{x^2 - a^2}}{x} + C$$

$$55. \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, \quad a \neq 0$$

$$56. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C, \quad a \neq 0$$

$$57. \int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$58. \int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$59. \int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$60. \int \sin ax \sin bxdx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\cos(a-b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$61. \int \sin ax \cos bxdx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2$$

$$62. \int \cos ax \cos bxdx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$63. \int \sin ax \cos ax dx = -\frac{\cos 2ax}{4a} + C, \quad a \neq 0$$

$$64. \int \sin^n ax \cos ax dx = \frac{\sin^{n+1} ax}{(n+1)a} + C, \quad a \neq 0, \quad n \neq -1$$

$$65. \int \cos^n ax \sin ax dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad a \neq 0, \quad n \neq -1$$

$$66. \int \frac{\sin ax}{\cos ax} dx = -\frac{1}{a} \ln|\cos ax| + C, \quad a \neq 0$$

$$67. \int \frac{\cos ax}{\sin ax} dx = \frac{1}{a} \ln|\sin ax| + C, \quad a \neq 0$$

$$68. \int \sin^n ax \cos^m ax dx$$

$$= -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax dx, \quad a \neq 0, \quad m+n \neq 0$$

$$69. \int \frac{dx}{b+c \sin ax} = \frac{-2}{a\sqrt{b^2-c^2}} \tan^{-1} \left| \sqrt{\frac{b-c}{b+c}} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) \right| + C, \quad a \neq 0, \quad b^2 > c^2$$

$$70. \int \frac{dx}{b+c \sin ax} = \frac{-1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \sin ax + \sqrt{c^2-b^2} \cos ax}{b+c \sin ax} \right| + C, \quad a \neq 0, \quad b^2 < c^2$$

$$71. \int \frac{dx}{1+\sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + C, \quad a \neq 0$$

$$72. \int \frac{dx}{1-\sin ax} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) + C, \quad a \neq 0$$

$$73. \int \frac{dx}{b+c \cos ax} = \frac{2}{a\sqrt{b^2-c^2}} \tan^{-1} \left| \sqrt{\frac{b-c}{b+c}} \tan \frac{ax}{2} \right| + C, \quad a \neq 0, \quad b^2 > c^2$$

$$74. \int \frac{dx}{b+c \cos ax} = \frac{1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \cos ax + \sqrt{c^2-b^2} \sin ax}{b+c \cos ax} \right| + C, \quad a \neq 0, \quad b^2 < c^2$$

$$75. \int \frac{dx}{1+\cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C, \quad a \neq 0$$

$$76. \int \frac{dx}{1-\cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C, \quad a \neq 0$$

$$77. \int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C, \quad a \neq 0$$

$$78. \int x^n \sin ax dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax dx, \quad a \neq 0$$

$$79. \int x^n \cos ax dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax dx, \quad a \neq 0$$

$$80. \int \tan ax dx = \frac{1}{a} \ln |\cos ax| + C, \quad a \neq 0$$

$$81. \int \cot ax dx = \frac{1}{a} \ln |\sin ax| + C, \quad a \neq 0$$

$$82. \int \tan^2 ax dx = \frac{1}{a} \tan ax - x + C, \quad a \neq 0$$

$$83. \int \cot^2 ax dx = -\frac{1}{a} \cot ax - x + C, \quad a \neq 0$$

$$84. \int \tan^n ax dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax dx, \quad a \neq 0, \quad n \neq 1$$

85. $\int \cot^n ax dx = -\frac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax dx, a \neq 0, n \neq 1$
86. $\int \sec ax dx = \frac{1}{a} \ln|\sec ax + \tan ax| + C, a \neq 0$
87. $\int \csc ax dx = -\frac{1}{a} \ln|\csc ax + \cot ax| + C, a \neq 0$
88. $\int \sec^n ax dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax dx, a \neq 0, n \neq 1$
89. $\int \csc^n ax dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} ax dx, a \neq 0, n \neq 1$
90. $\int \sec^n ax \tan ax dx = \frac{\sec^n ax}{na} + C, a \neq 0, n \neq 0$
91. $\int \csc^n ax \cot ax dx = -\frac{\csc^n ax}{na} + C, a \neq 0, n \neq 0$
92. $\int \sin^{-1} ax dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1-a^2 x^2} + C, a \neq 0$
93. $\int \cos^{-1} ax dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1-a^2 x^2} + C, a \neq 0$
94. $\int \tan^{-1} ax dx = x \tan^{-1} ax - \frac{1}{2a} \ln(1+a^2 x^2) + C, a \neq 0$
95. $\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax-1) + C, a \neq 0$
96. $\int b^{ax} dx = \frac{b^{ax}}{a \ln b} + C, a \neq 0, b > 0, b \neq 1$
97. $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx, a \neq 0$
98. $\int e^{ax} \sin bxdx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$
99. $\int e^{ax} \cos bxdx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$
100. $\int \ln ax dx = x \ln ax - x + C$
101. $\int x^n (\ln ax)^m dx = \frac{x^{n+1} (\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} dx, n \neq -1$
102. $\int \frac{(\ln ax)^m}{x} dx = \frac{(\ln ax)^{m+1}}{m+1} + C, m \neq -1$
103. $\int \frac{dx}{x \ln ax} = \ln|\ln ax| + C$